

A SPREADSHEET MODEL FOR TRANSPORTATION NETWORK DESIGN UNDER STOCHASTIC EQUILIBRIUM

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ABSTRACT. This study proposes a spreadsheet road network design (SRND) model for stochastic user equilibrium (SUE) transportation networks in urban areas. Road Network Design (RND) can be described as a numerical solution of the transportation problems which planers and traffic engineers face of. The RND also investigates how to manage current network capacity and to maintain scarce economic sources. For this purpose, a bi-level programming formulation for the RND is presented, in which the upper level model represents RND and a lower level represents road users' response. At the upper level problem, the design parameter is obtained using the quasi-Newton method and SUE link flows is obtained using Logit form of route choice probabilities at the lower level. The RND model is combined with the MathCAD program in order to find the derivatives of the link cost function. Finally, a solution algorithm for the SRND is proposed. Results showed that SRND model can be used for solving the RND problem in urban areas. It is also easy to apply since the derivatives of the path costs are obtained by internally calling the MathCAD program.

KEYWORDS: Road Network Design; Bi-Level Programming; Spreadsheets; Optimization

INTRODUCTION

Road network design (RND) problem can be described as a numerical solution of the transportation network problems which planers and traffic engineers face of. It investigates how

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to manage and to operate current transportation networks with available economic sources. RND may be carried out with effectively solving the traffic assignment and traffic control problem. Traffic control problem optimizes area traffic control parameters for fixed set of link traffic volumes for a short time period while traffic assignment problem obtains the equilibrium traffic volumes for fixed set of traffic control parameters for a short time period. The mutual interaction of these two problems can be explicitly considered, producing the so-called combined control and assignment problem.

A large number of network design problem (NDP) formulations have been proposed in the literature over the last decades, mostly based on nonlinear mathematical programming formulations. All existing NDP approaches assume that static traffic flow patterns on the improved network prevail. An extensive review of the NDP literature can be found in Yang and Bell (1998).

LeBlanc (1975) proposed a branch and bound approach for solving discrete NDPs. Dantzig et al. (1979) introduced a convex formulation assuming system optimum (SO) flow patterns and allowing continuous and discrete improvements; the solution algorithm was an extension to a decomposition approach introduced by Steenbrink (1974). Similarly, LeBlanc and Abdulaal (1979), Hoang (1982) and LeBlanc et al. (1984) suggested useful solution algorithms for NDP under the equilibrium of SO. On the other hand, user equilibrium (UE) based formulations are discussed in LeBlanc and Abdulaal and LeBlanc (1979), Marcotte (1983), LeBlanc and Boyce (1986), Suwansirikul et al. (1987), and Friesz et al. (1992). The solution methodologies are based on nonlinear optimization approaches, which could only handle small-size problems.

Abu-Lebdeh and Benekohal (2000) and Girianna and Benekohal (2001) presented formulations and solutions to control of oversaturated arterials and network control problems, respectively. Control was formulated as an optimization of dynamical problems and Micro-Genetic Algorithms (GA) were used with binary coding to optimize green splits and offsets. Ceylan and Bell (2004; 2005) used GAs to optimize signal timing with consideration to traffic assignment. Park et al. (2001) used mesoscopic simulation with a GA-based optimizer to simultaneously optimize all signal parameters with consideration to oversaturated conditions.

Gao et all (2004;2005) solved the discrete network design problem with the selection of link additions to an existing road network, with given demand from each origin to each destination. Chiou (2005) aimed to determine a continuous RND the set of link capacity expansions and corresponding equilibrium flows for which the measures of performance index for the network is optimal. A bi-level programming technique is used to formulate equilibrium network design problem. These two problems are formulated as bi-level programming problems with stochastic user equilibrium assignment as the second level programming problem. Lim et. all. (2005) formulated the continuous NDP for road expansion based on Stackelberg game where leader and follower exist, and allows for errors of travelers' behavior in choosing their routes. In order to solve the problem based on Stackelberg game, logit route choice model, in which there exists an explicit closed-form function between them, is used. The developed model will be applied to two example road networks for test and compared the results between the Stackelberg and Nash approaches to emphasize their difference between them.

All the existing solution methods mentioned so far uses analytical, numerical and heuristic algorithms to solve the RND. Solving teh RND with current methods may require a lengthy of mathematical formulations and solution procedures. But, there are no study solving the RND with a spreadsheet with an easy way by using spreadsheet facilities. Therefore, we proposed a spreadsheet road network design (SRND) model for stochastic equilibrium transportation networks in urban areas based on Stackelberg game.

The SRND model uses a bi-level formulation (Yang and Yagar, 1995) approach, where upper level solves the network design parameters on the spreadsheet and lower level solves stochastic equilibrium traffic assignment problem using Stackelberg game with quasi-Newton method via solver facility. At the lower level, the MathCAD program is internally called to obtain the derivatives of the equilibrium link flows. They are perturbed with sensitivity values of the link traffic volumes and the design parameters are obtained with every change on equilibrium flows on spreadsheets. During the solution of the lower level problem, logit route choice model is used. Sensitivity analysis algorithm is carried out to obtain the variations on perception parameter, θ . The perturbed flow is used to obtain the optimum or near-optimum values of design parameter.

PROBLEM FORMULATION

Let:

$G(N, L)$: A directed transportation road network
L	: Set of links
N	: Set of nodes.
$Z(\mathbf{q}, \mathbf{s})$: Network performance function
$\mathbf{t} = [t_w; \forall w \in \mathbf{W}]$: Vector of travel demand between each origin-destination pairs,
$\mathbf{W} = \{w = (i, sj; \forall i \in \mathbf{I}, \forall j \in \mathbf{J}$: Set of origin-destination pairs,
$\mathbf{q} = [q_a; \forall a \in \mathbf{L}]$: Vector of the average flow q_a on link a ,
$\mathbf{h} = [h_p; \forall p \in \mathbf{P}_w, \forall w \in \mathbf{W}]$: Vector of all path flows, where element h_p is traffic flow on path p .
$\mathbf{\delta} = [\delta_{ap}; \forall a \in \mathbf{L}, \forall p \in \mathbf{P}_w, \forall w \in \mathbf{W}]$: be the link/path incidence matrix, where $\delta_{ap} = 1$ if link a is on path p , and $\delta_{ap} = 0$ otherwise,
$\mathbf{\Lambda} = [\Lambda_{wp}; \forall p \in \mathbf{P}_w, \forall w \in \mathbf{W}]$: Origin-destination/path incidence matrix, where $\Lambda_{wp} = 1$ if path p connects origin-destination pair w , and $\Lambda_{wp} = 0$ otherwise,
$\mathbf{y} = [y_w; \forall w \in \mathbf{W}]$: Expected minimum origin-destination cost and summation is over all links.
$\mathbf{c}(\mathbf{q}, \mathbf{s}) = [c_a(q_a, \mathbf{s})]$: Vector of all link travel times, where element $c_a(q_a, \mathbf{s})$ is travel time on link a as a function of flow on the link itself and design parameters
\mathbf{s}	: Vector of feasible set of design parameters, $\mathbf{s} \in \mathbf{S}$
\mathbf{S}	: Feasible set of design parameters.

Bi-level programming formulation to obtain the design parameters, \mathbf{s} , and stochastic user equilibrium (SUE) link flows, $\mathbf{q}(\mathbf{s}^*)$ is

$$\underset{\mathbf{s}}{\text{Minimise}} \ Z(\mathbf{q}(\mathbf{s}^*), \mathbf{s}) = \sum_{a \in L} q_a(q_a, s) + y(s) \quad (1)$$

subject to $\mathbf{s} \in \mathbf{S}$

where $y(s)$ is the network improvement function and $(\mathbf{q}^*(\mathbf{s}), \mathbf{s})$ is obtained by solving the following optimization problem

$$\underset{q}{\text{Minimise}} \ Z_1(\mathbf{q}^*(\mathbf{s}), \mathbf{s}) = -\mathbf{t}^T \mathbf{y}(\mathbf{q}(\mathbf{s}), \mathbf{s}) + \mathbf{q}^T \mathbf{c}(\mathbf{q}(\mathbf{s}), \mathbf{s}) - \sum_{a \in L} \int_0^{q_a(s)} c_a(\mathbf{s}, x) dx \quad (2)$$

$$\text{subject to } \mathbf{t} = \mathbf{\Lambda} \mathbf{h}(\mathbf{s}), \quad \mathbf{q}(\mathbf{s}) = \mathbf{\delta} \mathbf{h}(\mathbf{s}), \quad \mathbf{h}(\mathbf{s}) \geq \mathbf{0}$$

The solution of (2) for fixed demand, \mathbf{t} , for O-D pairs w in W during a specified time period and the resulting equilibrium flows and corresponding travel times depend on design parameters. Thus, if any of the design parameters vary, the resulting equilibrium flows and corresponding travel times change.

The key for solving the bi-level programming model is to obtain the response function through solving the lower level problem and replace the design parameter, \mathbf{s} , in the upper level problem with the relationship between \mathbf{q} and \mathbf{s} - the response function. It connects the upper and lower level decision variables, which makes the two programming model dependent on each other.

In the Stackelberg game, we have the following linear approximate expression

$$q_a(s) = q_a(s^0) + \frac{\partial q_a(s^0)}{\partial s} (s - s^0) \quad (3)$$

where $s_0 \in \mathbf{S}$ is the initial value of design parameter. For solving the lower level given in (2), the following conditions must be hold:

$$Z_1(q_a^*(s), s) = 0 \text{ for any given } s \in \mathbf{S} \quad (4)$$

where $q_a^*(s)$ denotes the SUE link flow on link a . If we assume the function $Z_1(q_a, s)$ is to be twice differentiable, then the first-order expansion of $Z_1(q_a, s)$ in the neighborhood of $(q_a, s) = (q_a(s^0), s^0)$ is given as

$$Z_1(x, s) \approx Z_1(q_a^*(s^0), s^0) + \frac{\partial Z_1}{\partial q} \bigg|_{(q_a^*(s^0), s^0)} \bigg| + \frac{\partial Z_1}{\partial s} \bigg|_{(q_a^*(s^0), s^0)} \bigg| \quad (5)$$

where the derivative terms are the Jacobian matrices of $Z_l(q, s)$ with respect to q and s respectively, evaluated at $(q^*(s^0), s^0)$, which here denote J_q and J_s . Since $Z_l(q^*(s^0), s^0) = 0$ by SUE at s^0 , and we determine $s^0, q^*(s^0), J_q, J_s$, then for some other $s \neq s^0$, we can approximately solve the equilibrium condition $Z_l(q(s), s) = 0$ for $q(s)$ as

$$0 \approx 0 + J_q(q(s) - q^*(s^0)) + J_s(s - s^0) \quad (6)$$

By arranging expression (6)

$$q(s) = q^*(s^0) - J_q^{-1} \cdot J_s(s - s^0) \quad (7)$$

(7) is obtained. Sensitivity of equilibrium link flow with respect to design parameter is expressed in the form of the implicit function theorem as

$$\frac{\partial q}{\partial s} = -J_q^{-1} \cdot J_s$$

If we use the logit route choice model at the lower level, Equation (8) is written to obtain path, p_k , choice probabilities as:

$$p_k = \frac{\exp(\theta c_k)}{\sum_{i \in K} \exp(\theta c_i)} \quad (8)$$

where c_k is the route cost defined in (9) and θ is a road perception parameter of the error, K is path set for connecting each origin-destination pair, $w \in W$.

$$c_k = \sum_a c_a \delta_{ak} \quad (9)$$

c_a is a cost for link a and δ_{ak} is a dummy variable that 1 if the link a is on the route k , 0 otherwise. We have also a relation between p_k and link choice probability p_a as follows

$$p_a(c) = \sum_k p_k(c) \delta_{ak} \quad k \in K \quad (10)$$

Two derivatives of Z_l with respect to q and s are required in (7). The derivatives are obtained by internally calling the MathCAD program in the SRND model. Derivatives of the equilibrium link flows are then perturbed to obtain new SUE link flows and design parameters. The algorithmic steps of the SRND is in the following way.

Step 0 : $n = 0, s^0$ (Initialization)

Step 1 : $n = n + 1$

Step 2 : Solve the lower level problem with s^{n-1} with quasi-Newton method and yield $q(s^{n-1})$

Step 3 : Calculate derivative information with MATHCAD and yield $q(s^n, s^{n-1})$ by using following equation

$$q(s^n, s^{n-1}) = q(n^{-1}) - J_q^{-1} \dots J_s (s^n - s^{n-1})$$

Step 4 : Solve upper level problem with $q(s^n, s^{n-1})$ on spreadsheet and yield s^n

Step 5 : Convergence check, if criterion is met, stop; otherwise; go to Step 1.

A basic flowchart of the SRND model is given in Figure 1. It takes the initial values of design parameters and equilibrium link flows on spreadsheets and solves the upper level problem under SUE flow constraints and solves the lower level problem with quasi-Newton method. At the lower level, the derivatives and Jacobean matrix of the derivatives are obtained with MathCAD program. Then, “solver” function facility on spreadsheet solves the SUE traffic assignment problem. After that, the outputs are feed back to the upper level. The SRND model continues to the solution under converge criteria is met.

Solver can be used to maximize or minimize the value of a “target” worksheet cell by altering the values of other selected “changing” cells in the spreadsheet that influence the value in the target cell. It also allows constraints to be placed on the values of any cells in the worksheet. Thus, it is a general-purpose tool capable of solving constrained linear and nonlinear optimization problems. It may be called whenever it needs to obtain SUE link flows. Furthermore, some macro codes are written to internally call the MathCAD to obtain derivatives of SUE link flows with respect to design parameter.

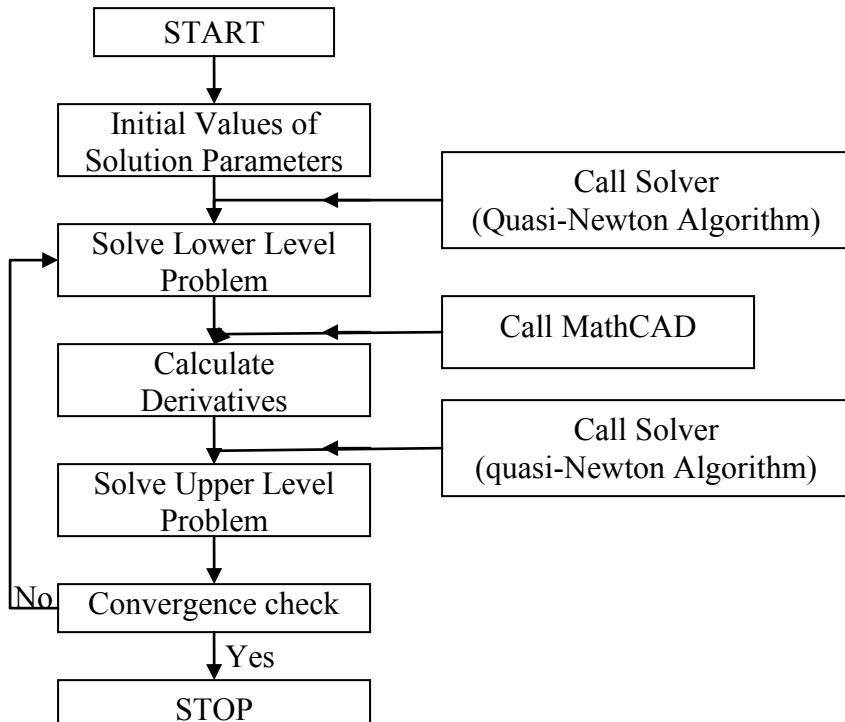


Figure 1. Flowchart of SRND model

In the solution process, the convergence criterion is set as 0.000001 that is the difference between current value of total network cost function and the previous one. Several convergence criteria may be used for stopping solution process. Illustrations of proposed SRND model with spreadsheet and MathCad model are applied to a simple network to test the model performance.

NUMERICAL CALCULATION WITH SRND MODEL

The chosen example involves a simple network with one O-D pair connecting 2 paths is given in Figure 2. Cost functions of both paths are a function of traffic volume and the design parameter s is only adopted on the link 1. In order to provide a better understanding the demand from node 1 to 2 is considered as 1 unit (1 vehicle/sec). Thus, cost functions of the links are given as:

$$c_1 = 1 + 2s x_1^2$$

$$c_2 = 2 + x_2$$

θ parameter represents the drivers' perception error on transportation network and it is set as 1 at the beginning state of SRND model application. Network construction cost is set to be

$$y(s) = 20(s^n - s^{n-1})^2$$

The equivalent path cost is expressed as:

$$Ec_a = c_a + \frac{1}{\theta} \ln(q_a) \quad (11)$$

By given cost functions and construction cost, the upper level objective function turns into

$$\underset{s}{\text{Minimise}} \quad Z(q(s^*), s, c(s)) = q_1(1 + 2s q_1^2) + q_2 * (2 + q_2) + 20(s^n - s^{n-1})^2 \quad (12)$$

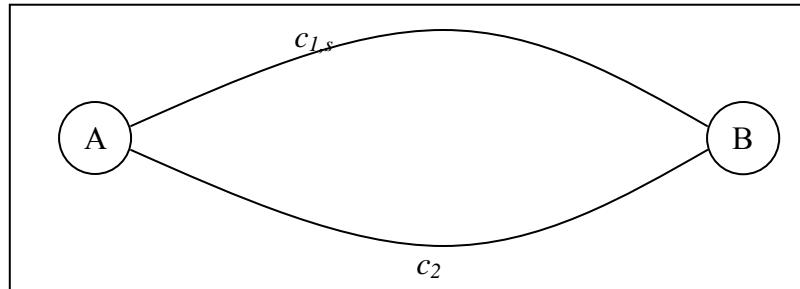


Figure 2. Example network

Equation (11) should satisfy the total flow condition $q_1 + q_2 = 1$. In order to solve the lower level problem by arranging (2), the following expressions are obtained as:

$$\frac{q_1}{1} = p_1 = \frac{e^{-\theta(1+2sq_1^2)}}{e^{-\theta(1+2sq_1^2)} + e^{-\theta(2+q_2)}} \text{ and } q_1 = \frac{1}{1 + e^{\theta(2sq_1^2 - q_2 - 1)}}$$

$$q_2 = \frac{1}{1 + e^{-\theta(2sq_1^2 - q_2 - 1)}}$$

$$Z_1(q_1, q_2, s) = q_1 + q_1 \cdot e^{\theta(2sq_1^2 - q_2 - 1)} - 1 \quad (13)$$

$$Z_1(q_1, q_2, s) = q_2 + q_2 \cdot e^{-\theta(2sq_1^2 - q_2 - 1)} - 1 \quad (14)$$

SUE link traffic volumes for the link 1 and link 2 (q_1 and q_2) are calculated using Equation (13) and (14) by iterations.

Jacobian matrix of the derivatives are

$$J_q = \begin{bmatrix} \frac{\partial Z_{11}}{\partial q_1} & \frac{\partial Z_{11}}{\partial x_2} \\ \frac{\partial Z_{12}}{\partial q_1} & \frac{\partial Z_{12}}{\partial q_2} \end{bmatrix}; J_s = \begin{bmatrix} \frac{\partial Z_{11}}{\partial s} \\ \frac{\partial Z_{12}}{\partial s} \end{bmatrix}$$

Traffic volumes are obtained by applying quasi-Newton method on spreadsheet solver menu for given $s=1$.

$$x_1^* = 0.6356 \quad x_2^* = 0.3644$$

By using Step 3, the new perturbed traffic volumes are calculated with (7) as:

$$x_1(s') = 0.588 \quad x_2(s') = 0.412$$

After that the full application of SRND model is carried out in an example network. The flow diagram of the model in detail is given in Figure 3 until solution is found.

A sample spreadsheet is given on Figure 4. The values on the sheet denote the iteration number and constraints given by the user and the constraints that have to be satisfied by all of the parameters of the design problem. The traffic volumes under SUE assumption at the lower level program are calculated by macro after clicking the *Start Process Button*. Then the Jacobean matrices are determined by calculating derivative of the cost functions by internally calling the MathCAD program. Traffic volumes are perturbed with $J_q^{-1} * J_s$. After determining the new values, solver window is called and the upper level problem, which satisfied the traffic volume constraints is solved. This process continued until convergence criterion is satisfied. In this study, following stopping criteria is used.

$$\frac{Abs(Z^n - Z^{n-1})}{Z^{n-1}} \leq 0.000001$$

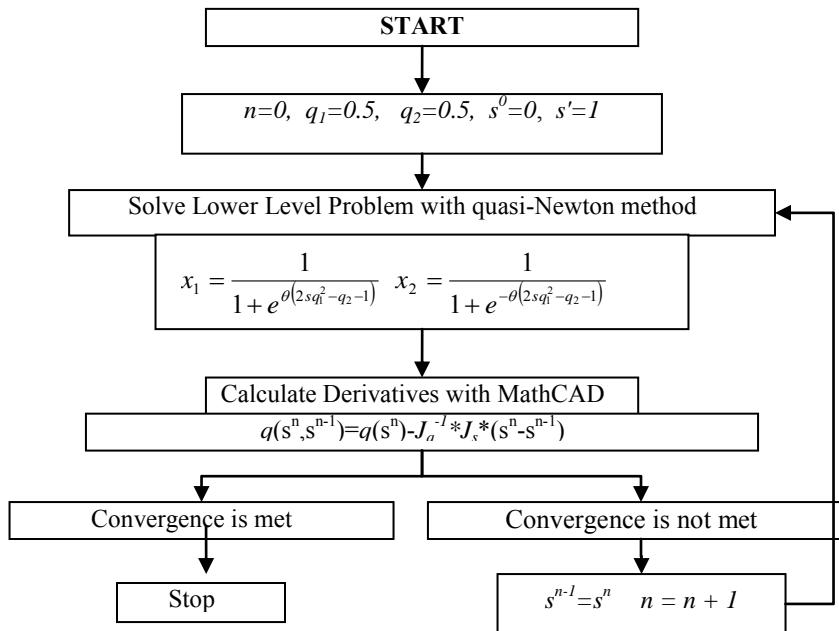


Figure 3. Flow diagram of the SRND model

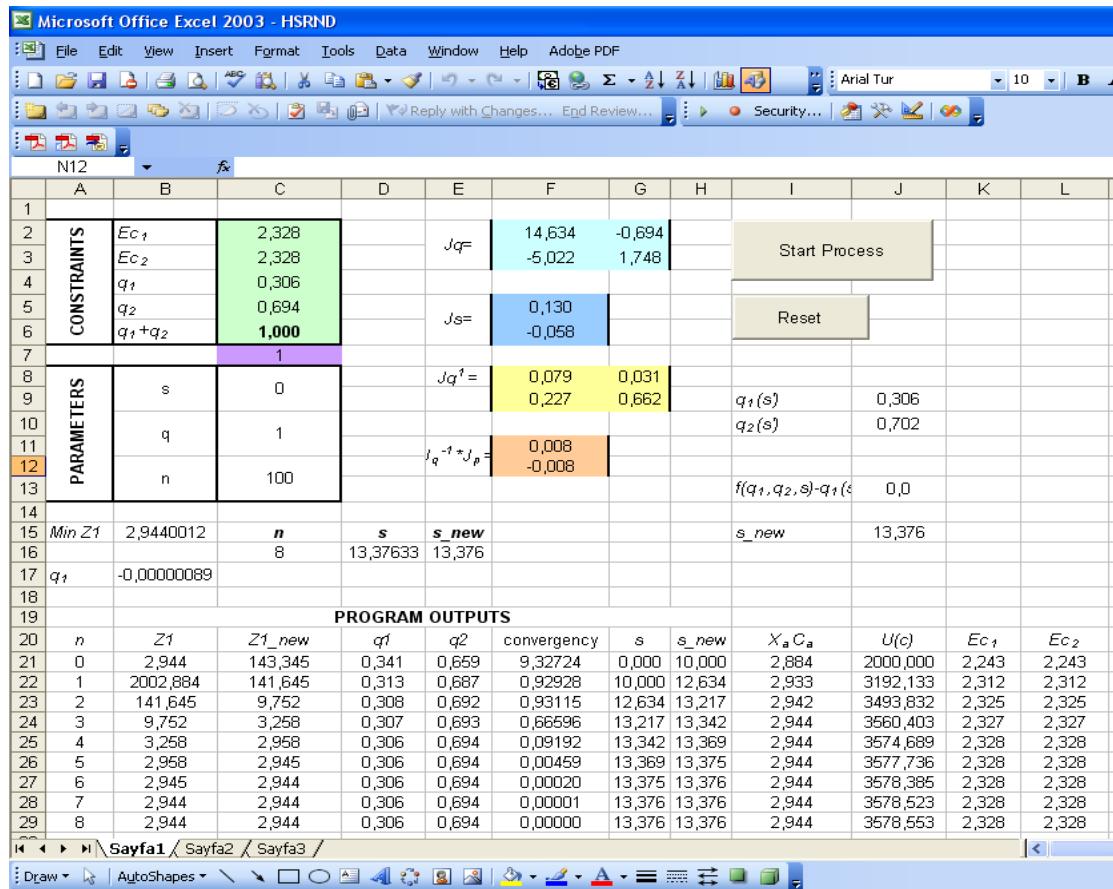


Figure 4. Solution of the NDP using Spreadsheets

The application of SRND model outputs and parameter sensitivities of the example transport network is given in Table 1, where n is step number, q_1 and q_2 are traffic volumes, $y(s)$ is the construction cost, Ec_1 and Ec_2 are the equivalent paths costs and the last column includes the value of total network cost. It is unavoidable for the $y(s)$ to grow up throughout the iterations due to non-convex property of the problem. As can be seen in Table 1, the equivalent path costs are equal at the last two columns, but the network investment cost increase for every iteration which shows a costly structure of the investments and Braess paradox (Sheffi, 1985).

The application of SRND model is carried out to obtain the convergence of the algorithm and the optimal or near optimal values of design parameter. The convergence behavior of the model is given in Figure 5. The algorithm reaches the optimum or near optimum value by about 7 iterations.

The change of the total network cost can be seen in Figure 6. The total network cost is about 22 units at the first iteration and it is improved to a level of about 4 units that indicates about 80% improvement after 17 iteration.

Table 1. Solution of the NDP on the sample network

n	x_1	x_2	s	$y(s)$	Ec_1	Ec_2
0	0.636	0.364	1.000	20.000	1.355	1.355
1	0.588	0.412	1.526	46.568	1.525	1.525
2	0.570	0.430	1.772	62.786	1.587	1.587
3	0.562	0.438	1.881	70.789	1.612	1.612
4	0.559	0.441	1.929	74.432	1.623	1.623
5	0.557	0.443	1.950	76.037	1.627	1.627
6	0.557	0.443	1.959	76.734	1.629	1.629
7	0.557	0.443	1.963	77.035	1.630	1.630
8	0.556	0.444	1.964	77.164	1.630	1.630
9	0.556	0.444	1.965	77.220	1.631	1.631
10	0.556	0.444	1.965	77.245	1.631	1.631
11	0.556	0.444	1.965	77.256	1.631	1.631
12	0.556	0.444	1.965	77.261	1.631	1.631
13	0.556	0.444	1.966	77.264	1.631	1.631
14	0.556	0.444	1.966	77.266	1.631	1.631
15	0.556	0.444	1.966	77.267	1.631	1.631
16	0.556	0.444	1.966	77.268	1.631	1.631
17	0.556	0.444	1.966	77.268	1.631	1.631

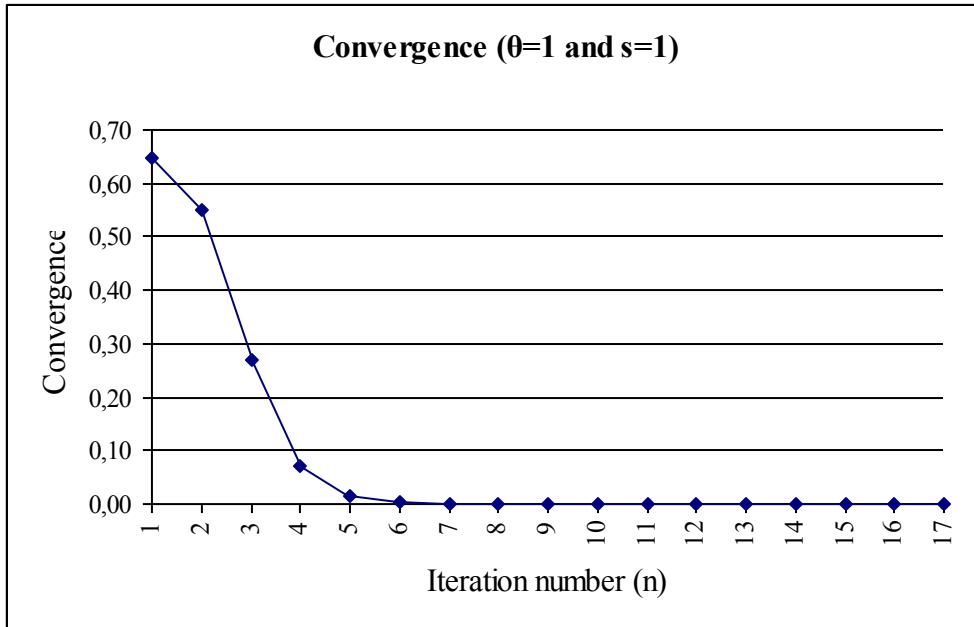


Figure 5. Convergence graph

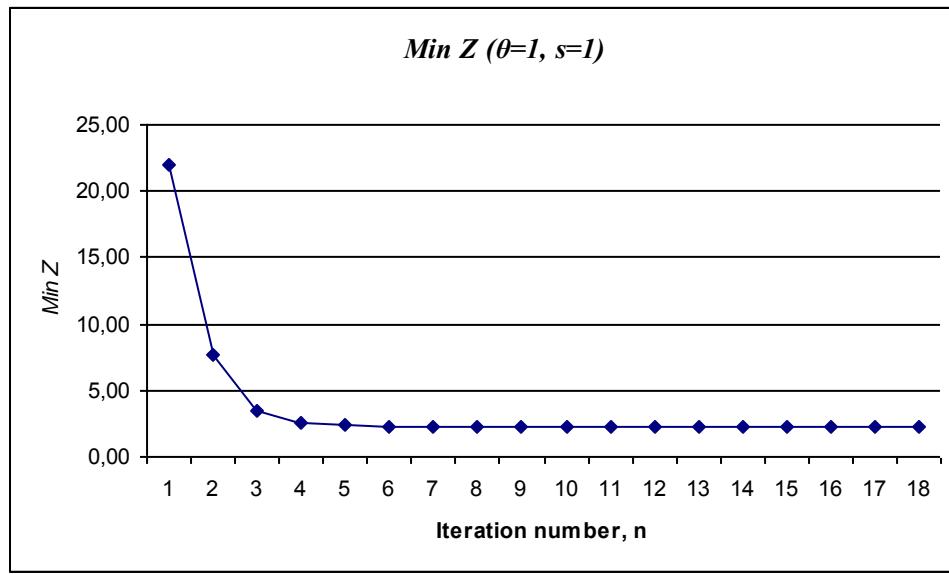


Figure 6. Change of the total cost function values by iterations

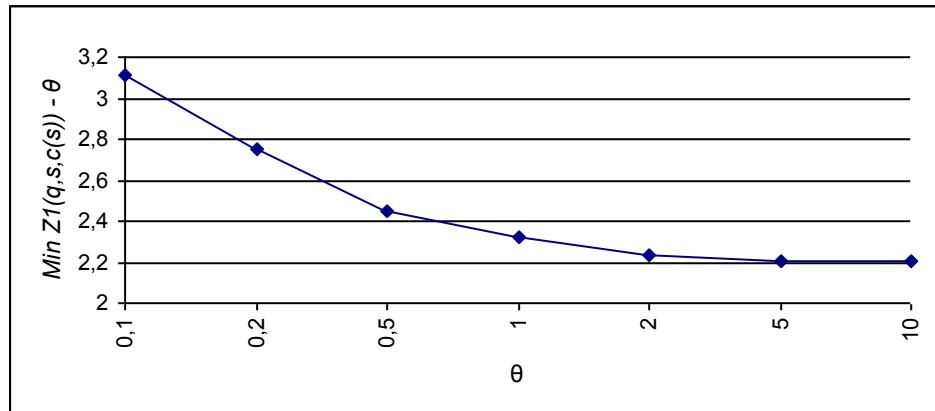
SENSITIVITY ANALYSIS OF THE SRND MODEL

Due to the non-convex nature of the RND and many parameters that need to be checked, it is therefore required to analyze the sensitivities of the SRND model parameters. The SRND model algorithm is checked for the different values of θ and s . Change of the traffic volumes and cost function values due to the different values of θ and design parameter s is given in Table 2. If θ increases the total network costs will decrease and if it decreases the network costs will increase. The change of the cost function value for different values of θ is given in Figure 7. Road network become stochastic when θ is less then 1, and the network will tend to reach the user equilibrium when θ is bigger then 1. This can be easily seen from Figure 7.

Table 2. Change of the traffic volumes and cost function values for different values of θ

θ	q_1	q_2	s	$\min Z_l[q, s, c(s)]$
0.1	0.469	0.531	6.289	3.111
0.2	0.477	0.523	4.390	2.747
0.5	0.509	0.491	2.731	2.453
1	0.556	0.444	1.966	2.318
2	0.616	0.384	1.513	2.239
15	0.689	0.311	1.214	2.202
10	0.728	0.272	1.108	2.201

The change of traffic volumes and total network costs for given different values of design parameter s is given in Table 3. If the design parameter increases, where the new links may be constructed, total network cost increases.

**Figure 7.** Change of the network cost function value for different θ values.**Table 3.** Sensitivities of different design parameters

s	x_1	x_2	$\min Z_l[q, s, c(s)]$
0.1	0.709	0.291	1.658
0.2	0.676	0.324	1.826
0.5	0.616	0.384	2.094
0.6	0.601	0.399	2.151
1	0.556	0.444	2.318
2	0.485	0.515	2.543
3	0.440	0.560	2.666
5	0.382	0.618	2.802
10	0.306	0.694	2.944

CONCLUSIONS

The SRND model uses a bi-level formulation approach, where upper level solves the network design parameters on the spreadsheet and lower level solves stochastic equilibrium traffic assignment problem using Stackelberg game using quasi-Newton method on using solver facility. At the lower level, the MathCAD program is internally called to obtain the derivatives of the equilibrium link flows. Equilibrium link flows are perturbed with sensitivity values of the link traffic volumes and the design parameters are obtained with every change on equilibrium flows on spreadsheets. The flowchart of the HSRN model and corresponding convergence behavior is given. During the solution of the lower level problem, logit route choice model is used. Sensitivity analysis algorithm is carried out to obtain on the variations on perception parameter, θ . The perturbed flow is used to obtain the optimum or near-optimum values of design parameter. Following conclusions may be drawn from this study.

The SRND model may be applied to solve the bi-level RND problem by using spreadsheet under the assumption of Stackelberg game. The SUE link flows may be obtained by applying the quasi-Newton method. Derivatives of the link and path cost functions may be calculated by internally calling the MathCAD program. The SRND model may provide visualization for transport network designers. It also obtained that the steady convergence may be achieved if RND problem is solved with spreadsheet.

Sensitivity analysis of the perception parameter on path costs showed that if it decreases road networks users become stochastic that leads to SUE and if it increases the network leads to the user equilibrium.

The sensitivity of the design parameters, s , a new link added to a network, leads to Braess paradox, which increases the total network cost.

The proposed SRND model is applied in a very simple network to show its performance on a spreadsheet. It would be better to apply the model to a realistic size of network, but it may not affect the overall aim of this study since our aim is to solve the RND on spreadsheets. Future studies will be on the application of the SRND model for realistic size of transportation networks for various sets of design parameters.

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