MAT 219 DİFERANSİYEL DENKLEMLER (ARA SINAV)

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Adı Soyadı		1(25)	2(25)	3(25)	4(25)	Toplam
No						

1) $\left(e^x + A\frac{\sin x}{y}\right)dx + \left[B\frac{\cos x}{y^2} + (B-1)e^x\right]dy = 0 \quad , \quad A, B:$ sabit

diferansiyel denklemini tam diferansiyel yapan A ve B sabitlerini bulunuz ve çözünüz.

2) Aşağıdaki diferansiyel denklemin genel çözümünü bulunuz.

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx}$$

3) $\cos y = c\,e^{-x}$ eğri ailesinin dik yörüngelerini bulunuz.

4) Aşağıdaki diferansiyel denklemin genel çözümünü bulunuz.

$$\frac{dy}{dx} + \frac{1}{x}y = -2x^5y^4$$

Süre: 90 dak. Başarılar

Dersi veren öğretim üyesi:

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(1) $\underbrace{\left(e^x + A\frac{\sin x}{y}\right)}_{M}dx + \underbrace{\left[B\frac{\cos x}{y^2} + (B-1)e^x\right]}_{N}dy = 0$

$M_y = -\frac{A\sin x}{y^2} = N_x = -\frac{B\sin x}{y^2} + (B-1)e^x$

$A = B$ ve $(B-1) = 0 \Rightarrow A = 1, B = 1$

$\left(e^x + \frac{\sin x}{y}\right)dx + \frac{\cos x}{y^2}dy = 0$

$\frac{\partial \phi}{\partial x} = e^x + \frac{\sin x}{y}$, $\frac{\partial \phi}{\partial y} = \frac{\cos x}{y^2}$

$\downarrow$

$\phi(x,y) = e^x - \frac{\cos x}{y} + h(y)$

$\frac{\partial \phi}{\partial y} = \cancel{\frac{\cos x}{y^2}} + h'(y) = \cancel{\frac{\cos x}{y^2}} \Rightarrow h'(y) = 0$

$h(y) = c_1$

$\phi(x,y) = c$

$e^x - \frac{\cos x}{y} + c_1 = c$

$$\boxed{e^x - \frac{\cos x}{y} = c_2}$$

(2) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)$

$\frac{dy}{dx} = u$ diyelim $\frac{d^2y}{dx^2} = \frac{du}{dx}$ olur.

$\frac{du}{dx} = u^2 + 2u \Rightarrow \frac{du}{u(u+2)} = dx$

$\int \frac{du}{u(u+2)} = \int dx \Rightarrow \frac{1}{2}\int \frac{du}{u} - \frac{1}{2}\int \frac{du}{u+2} = \int dx$

$\frac{1}{2}\ln|u| - \frac{1}{2}\ln|u+2| - \frac{1}{2}\ln|c_1| = x$

$\frac{1}{2}\ln|u| - \frac{1}{2}\ln|c_1(u+2)| = x$

$\frac{1}{2}\ln\left|\frac{u}{c_1(u+2)}\right| = x \Rightarrow \frac{u}{c_1(u+2)} = e^{2x}$

$\frac{u}{u+2} = c_1 e^{2x} \Rightarrow \frac{dy}{dx} = c_1 e^{2x}\frac{dy}{dx} + 2c_1 e^{2x}$

$\frac{dy}{dx} = \frac{2c_1 e^{2x}}{1 - c_1 e^{2x}} \Rightarrow \int dy = \int \frac{2c_1 e^{2x}}{1 - c_1 e^{2x}} dx$

$y = -\ln|1 - c_1 e^{2x}| + \ln|c_2|$

$$y = \ln\left|\frac{c_2}{1 - c_1 e^{2x}}\right|$$

(3) $\cos y = c e^{-x} \Rightarrow c = e^{x}\cos y$

$-\frac{dy}{dx}\cdot \sin y = -c e^{-x} \Rightarrow$ ~~$\sin y\, dy = c e^{x} dx$~~

$\frac{dy}{dx} = c\frac{e^{-x}}{\sin y} = \frac{e^{x}\cos y \cdot e^{-x}}{\sin y} = \frac{\cos y}{\sin y}$

$\frac{dy}{dx} = \frac{\cos y}{\sin y} \Rightarrow \boxed{\frac{dy}{dx} = -\frac{\sin y}{\cos y}}$ (12)

bulunaşalı eğri ailesinin dif. denk.

$\int \frac{\cos y}{\sin y} dy = -\int dx$

$\ln|\sin y| = -x + \ln|c_1|$

$\boxed{\sin y = c_1 e^{-x}}$ (13)

(4) $\frac{dy}{dx} + \frac{1}{x}y = -2x^5 y^4$ Bernoulli

$u = y^{1-4} \Rightarrow u = y^{-3}$ $\quad u^{-4/3}$

$\frac{du}{dx} = -3y^{-4}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{1}{3}y^4\frac{du}{dx}$

$-3u^{4/3}/ \quad -\frac{1}{3}u^{-4/3}\frac{du}{dx} + \frac{1}{x}u^{-1/3} = -2x^5 u^{-4/3}$

$\frac{du}{dx} - \frac{3}{x}u = 6x^5$ (12) lineer

$I(x) = e^{-3\int\frac{dx}{x}} = e^{-3\ln x} = x^{-3}$

$\frac{d}{dx}\left[x^{-3}u\right] = 6x^2 \Rightarrow x^{-3}u = 2x^3 + C$

$u = 2x^6 + cx^3$

$y^{-3} = 2x^6 + cx^3$

$y = \left(\frac{1}{2x^6 + cx^3}\right)^{1/3}$ (13)