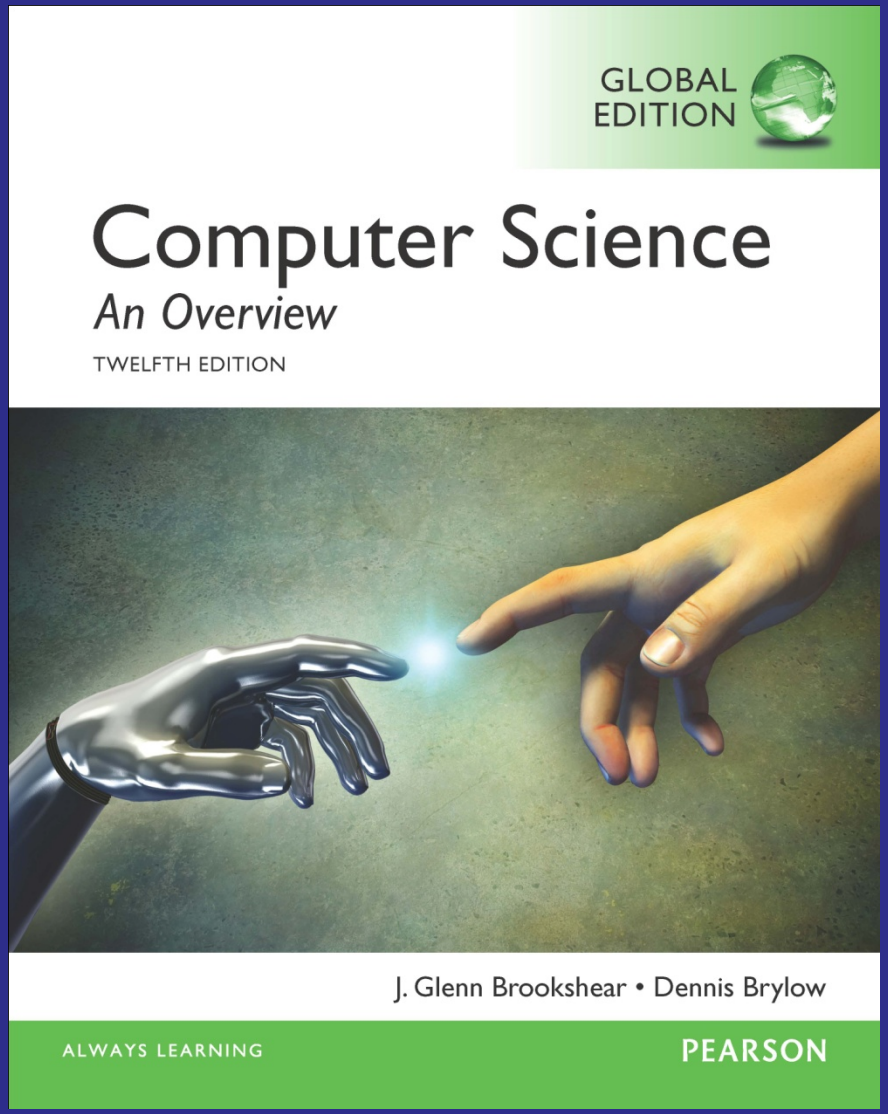
# Chapter 12: Theory of Computation

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# Chapter 12: Theory of Computation

- 12.1 Functions and Their Computation
- 12.2 Turing Machines
- 12.3 Universal Programming Languages
- 12.4 A Noncomputable Function
- 12.5 Complexity of Problems
- 12.6 Public-Key Cryptography

# Functions

- **Function:** A correspondence between a collection of possible input values and a collection of possible output values so that each possible input is assigned a single output (in mathematical sense)

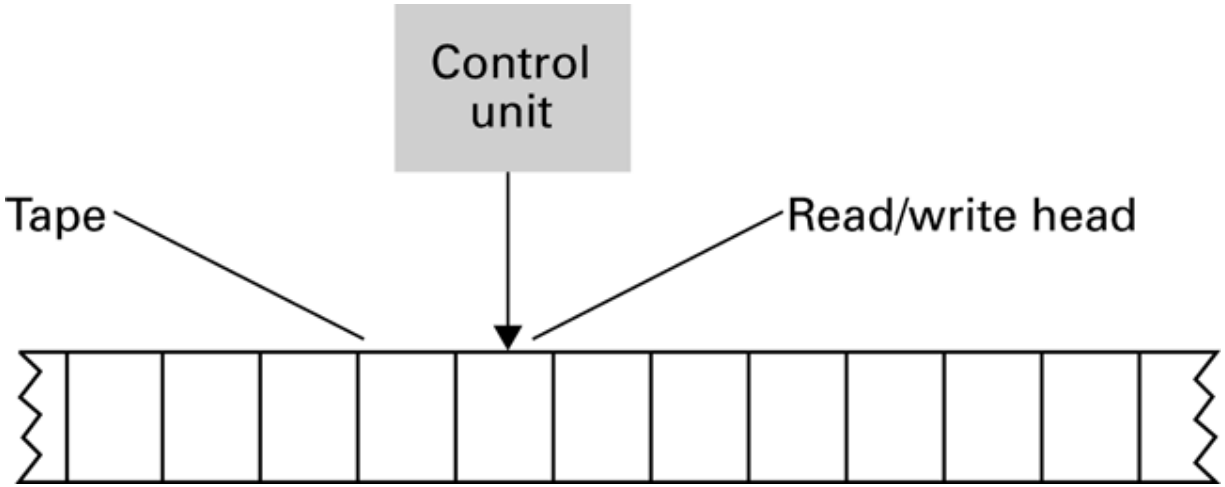
# Functions (continued)

- **Computing a function:** Determining the output value associated with a given set of input values
- **Noncomputable function:** A function that cannot be computed by any algorithm

# Figure 12.1 An attempt to display the function that converts measurements in yards into meters

| Yards (input) | Meters (output) |
|---|---|
| 1 | 0.9144 |
| 2 | 1.8288 |
| 3 | 2.7432 |
| 4 | 3.6576 |
| 5 | 4.5720 |
| . | . |
| . | . |
| . | . |

# Figure 12.2 **The components of a Turing machine**



# Turing Machine Operation

- Inputs at each step
  - State
  - Value at current tape position
- Actions at each step
  - Write a value at current tape position
  - Move read/write head
  - Change state

# Figure 12.3 **A Turing machine for incrementing a value**

| Current state | Current cell content | Value to write | Direction to move | New state to enter |
|---|---|---|---|---|
| START | * | * | Left | ADD |
| ADD | 0 | 1 | Right | RETURN |
| ADD | 1 | 0 | Left | CARRY |
| ADD | * | * | Right | HALT |
| CARRY | 0 | 1 | Right | RETURN |
| CARRY | 1 | 0 | Left | CARRY |
| CARRY | * | 1 | Left | OVERFLOW |
| OVERFLOW | (Ignored) | * | Right | RETURN |
| RETURN | 1 | 0 | Right | RETURN |
| RETURN | * | 1 | Right | RETURN |
| RETURN | | * | No move | HALT |

# Church-Turing Thesis

- The functions that are computable by a Turing machine are exactly the functions that can be computed by any algorithmic means

# Universal Programming Language

- A language with which a solution to any computable function can be expressed
  - Examples: “Bare Bones” and most popular programming languages

# The Bare Bones Language

- Bare Bones is a simple, yet universal language
- Statements

```
clear name
incr name
decr name
while name not 0:
        .
        .
        .
```

# Figure 12.4 **A Bare Bones program for computing X x Y**

```
clear Z
while X not 0:
    clear W
    while Y not 0:
        incr Z
        incr W
        decr Y
    while W not 0:
        incr Y
        decr W
    decr X
```

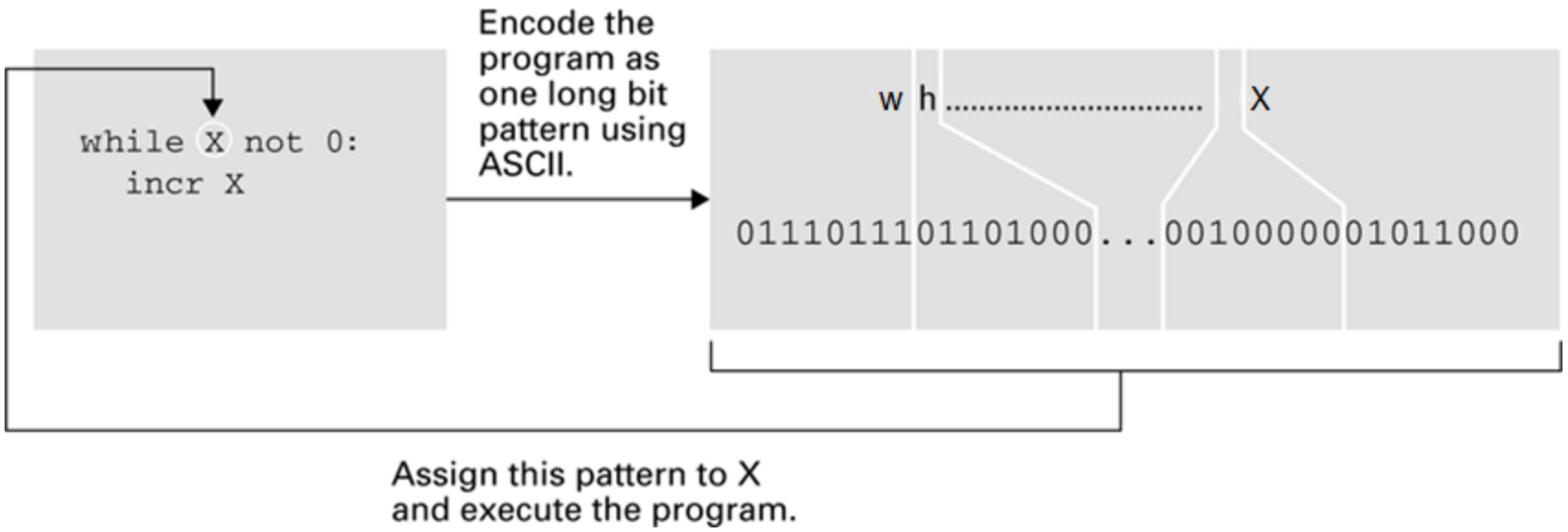
# Figure 12.5 A Bare Bones implementation of the instruction "copy Today to Tomorrow"

```
clear Aux
clear Tomorrow
while Today not 0:
    incr Aux
    decr Today
while Aux not 0:
    incr Today
    incr Tomorrow
    decr Aux
```

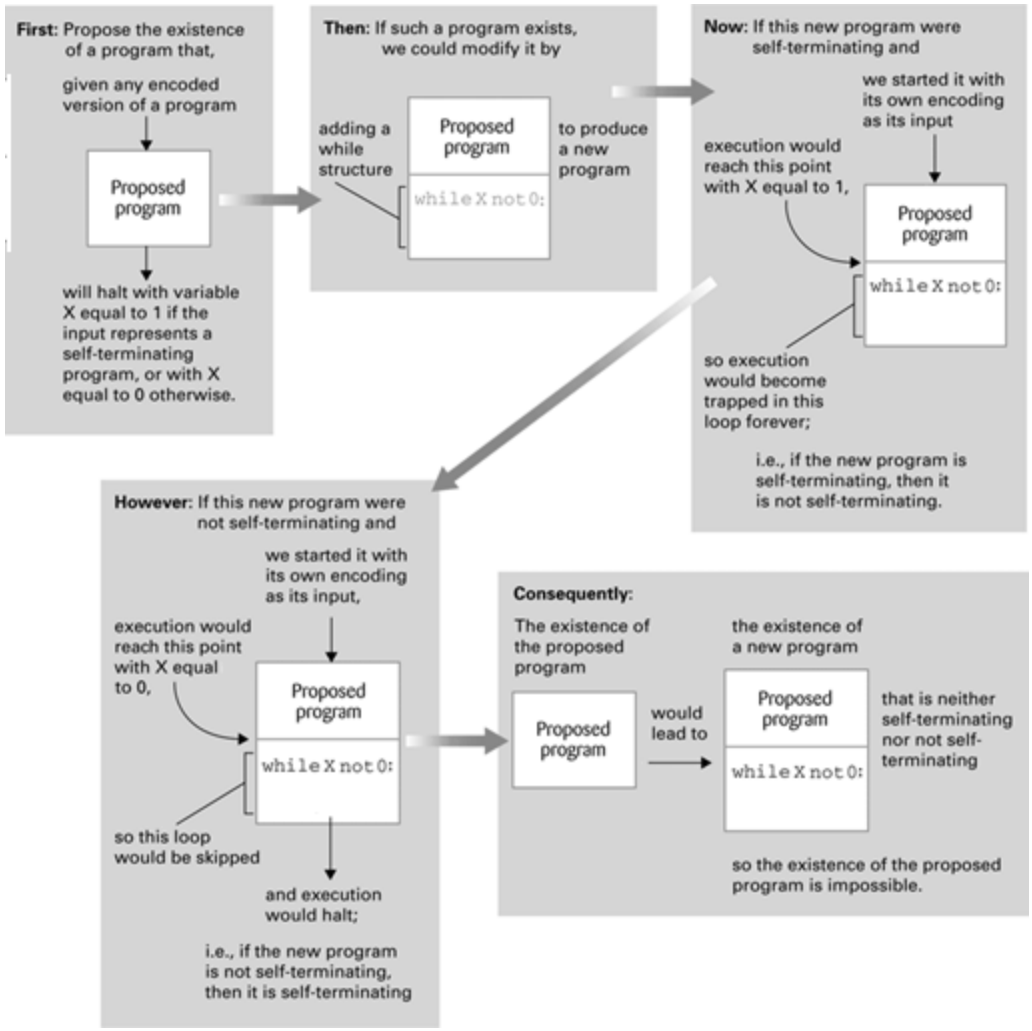
# The Halting Problem

- Given the encoded version of any program, return 1 if the program is self-terminating, or 0 if the program is not.

# Figure 12.6 **Testing a program for self-termination**



# Figure 12.7 **Proving the unsolvability of the halting program**



# Complexity of Problems

- **Time Complexity:** The number of instruction executions required
  - Unless otherwise noted, “complexity” means “time complexity.”
- A problem is in class O(f(n)) if it can be solved by an algorithm in $\Theta(f(n))$.
- A problem is in class $\Theta(f(n))$ if the best algorithm to solve it is in class $\Theta(f(n))$.

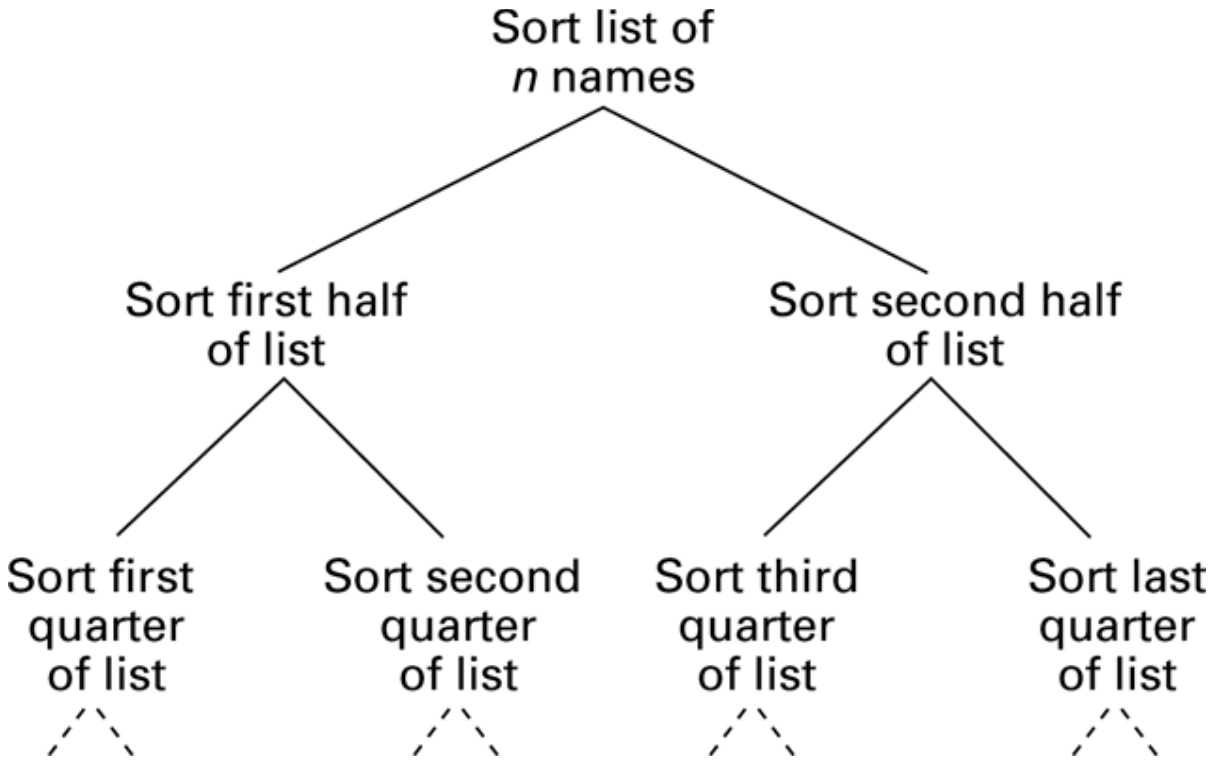
# Figure 12.8 **MergeLists function for merging two lists**

```
def MergeLists (InputListA, InputListB, OutputList):
    if (both input lists are empty):
        Stop, with OutputList empty
    if (InputListA is empty):
        Declare it to be exhausted
    else:
        Declare its first entry to be its current entry
    if (InputListB is empty):
        Declare it to be exhausted
    else:
        Declare its first entry to be its current entry
    while (neither input list is exhausted):
        Put the "smaller" current entry in OutputList.
        if (that current entry is the last entry in its corresponding input list):
            Declare that input list to be exhausted
        else:
            Declare the next entry in that input list to be the list's current entry
    Starting with the current entry in the input list that is not exhausted,
    copy the remaining entries to OutputList
```

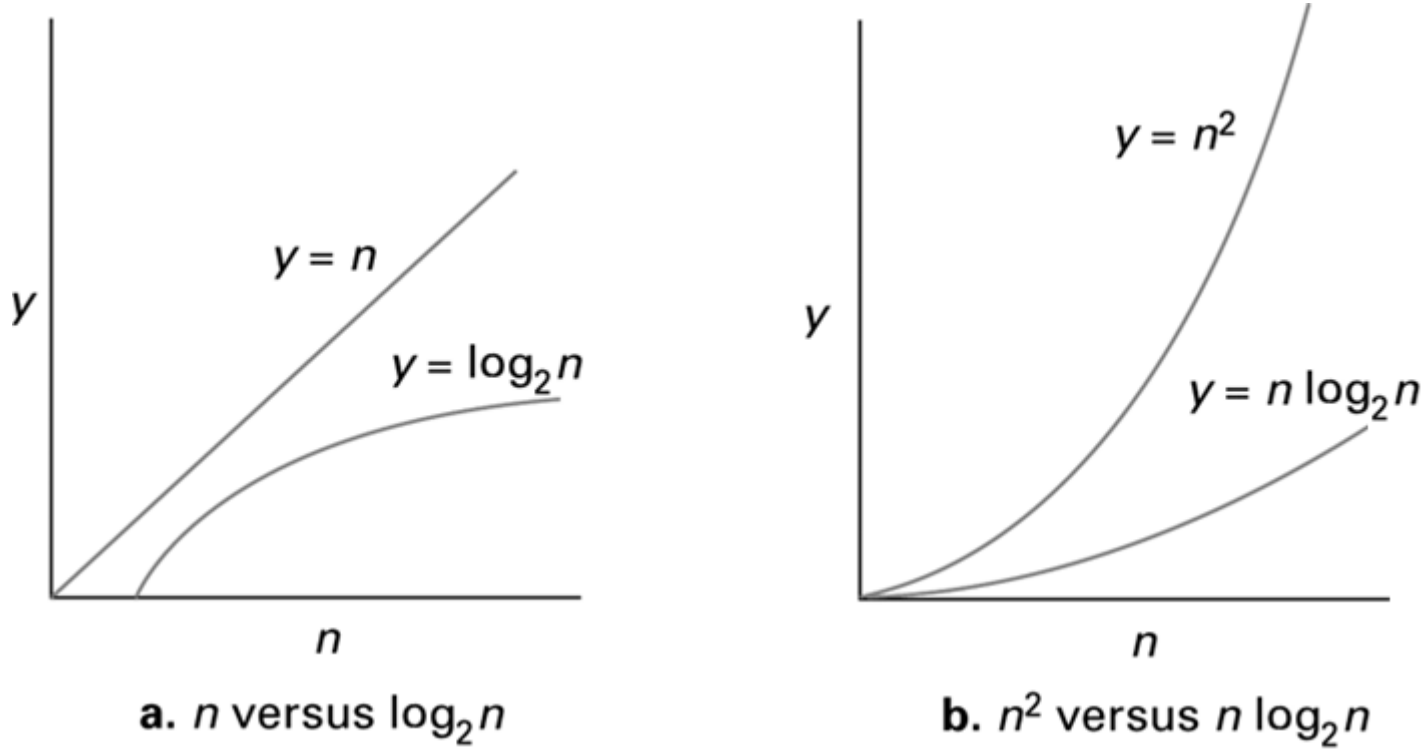
# Figure 12.9 **The merge sort algorithm implemented as a function MergeSort**

```
def MergeSort (List):
    if (List has more than one entry):
        MergeSort(the first half of List)
        MergeSort(the second half of List)
        MergeLists(first and second halves of List)
```

# Figure 12.10 **The hierarchy of problems generated by the merge sort algorithm**



# Figure 12.11 Graphs of the mathematical expressions $n$, $\log_2 n$, $n \log_2 n$, and $n^2$




**a.** $n$ versus $\log_2 n$

**b.** $n^2$ versus $n \log_2 n$

# P versus NP

- **Class P:** All problems in any class $\Theta(f(n))$, where f(n) is a polynomial
- **Class NP:** All problems that can be solved by a nondeterministic algorithm in polynomial time

  **Nondeterministic algorithm** = an "algorithm" whose steps may not be uniquely and completely determined by the process state
- Whether the class NP is bigger than class P is currently unknown.

# Traveling Salesperson

- **Involves traveling salesperson** who must visit of each of her clients in different cities without exceeding her travel budget.

- **The traditional solution to this problem** is to consider the potential paths in a systematic manner, comparing the leanth of each path to the miliage limit until either an acceptable path is found or all posibilities have been considered.

# Traveling Salesperson

Pick one of the possible paths, and compute its total distance.

if (this distance is not greater than the allowable mileage)):
    Declare a success.
else:
    Declare nothing.

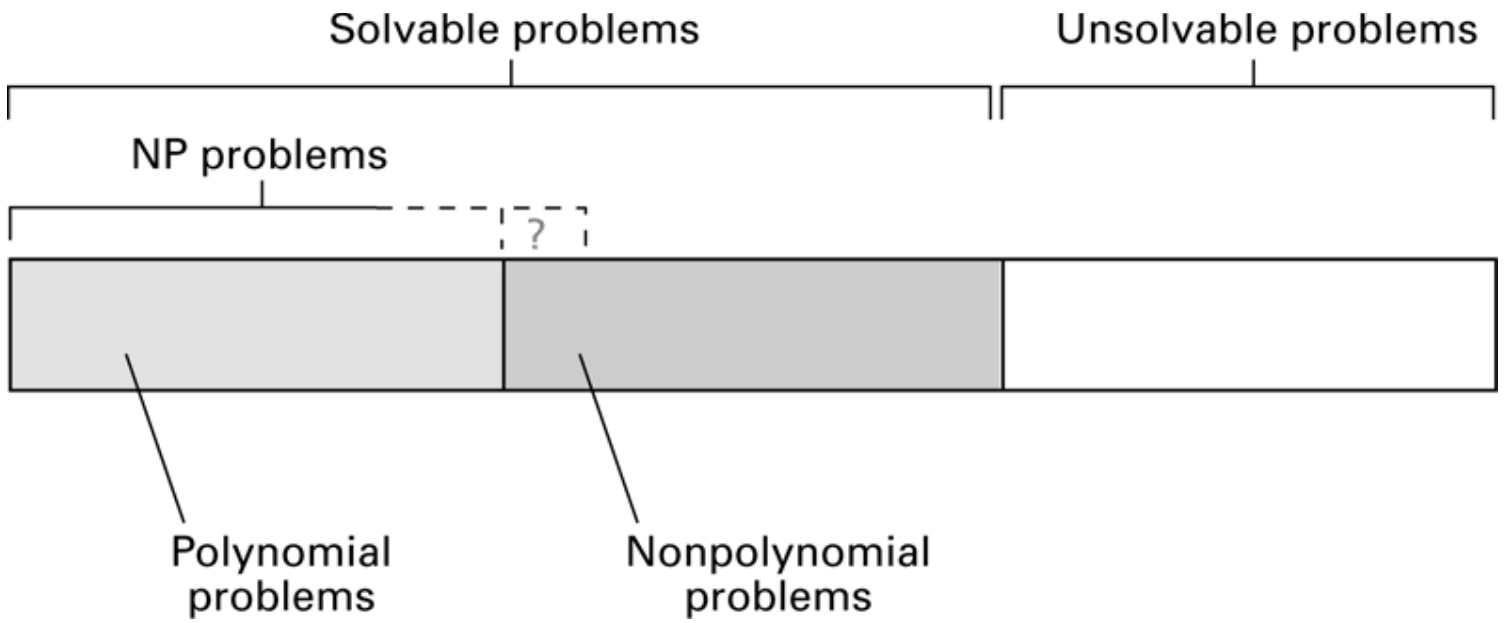
# NP problem

A problem that can be solved in polynomial time by a nondeterministic algorithm is called a **nondeterministic polynomial problem.**

**Note that all problems in P are also in NP.**

**Whether all of the NP problems are also in P, is an open question**

# Figure 12.12 **A graphic summation of the problem classification**



# Public-Key Cryptography

- **Key:** A value used to encrypt or decrypt a message
  - **Public key**: Used to encrypt messages
  - **Private key**: Used to decrypt messages
- **RSA:** A popular public key cryptographic algorithm (Ron Rivest, Adi Shamir, Len Adleman)
  - Relies on the (presumed) intractability of the problem of factoring large numbers

# Public-Key Cryptography

- 
- *Mathematics tells us thatif p and q are prime numbers and m is an integer between 0 and pg then*
- $1 = m^{k(p-1)(q-1)}$ %pq

# Public-Key Cryptography

- 
- $exd = k(p-1)(q-1)+1$
- For some positive integer k
- p,q,n,e,d
- e,n are the encyription keys
- d,n are decryption keys.

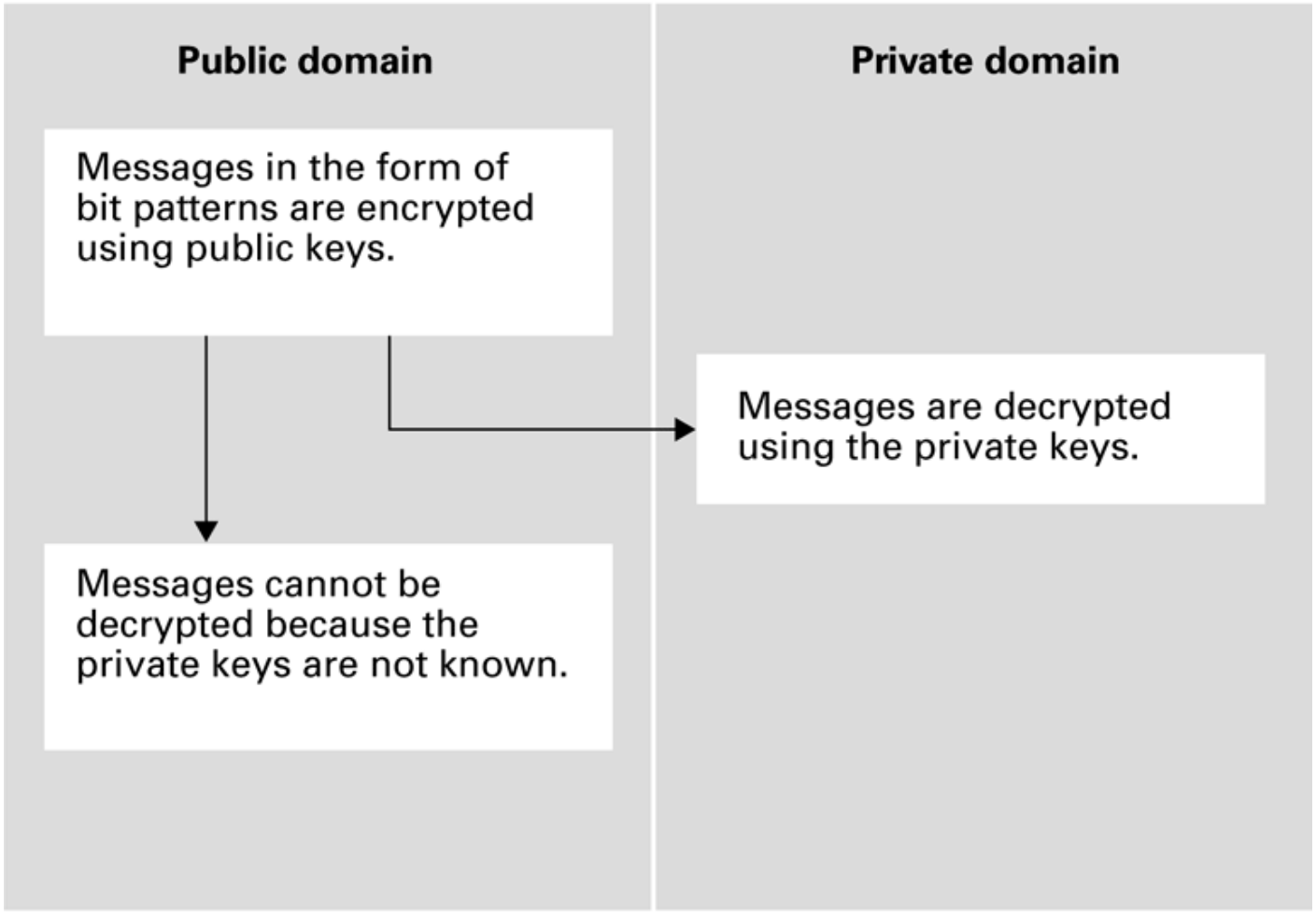
# Encrypting the Message 10111

- Encrypting keys: n = 91 and e = 5
- $10111_{two} = 23_{ten}$
- $23^{e} = 23^{5} = 6{,}436{,}343$
- 6,436,343 ÷ 91 has a remainder of 4
- $4_{ten} = 100_{two}$
- Therefore, encrypted version of 10111 is 100.

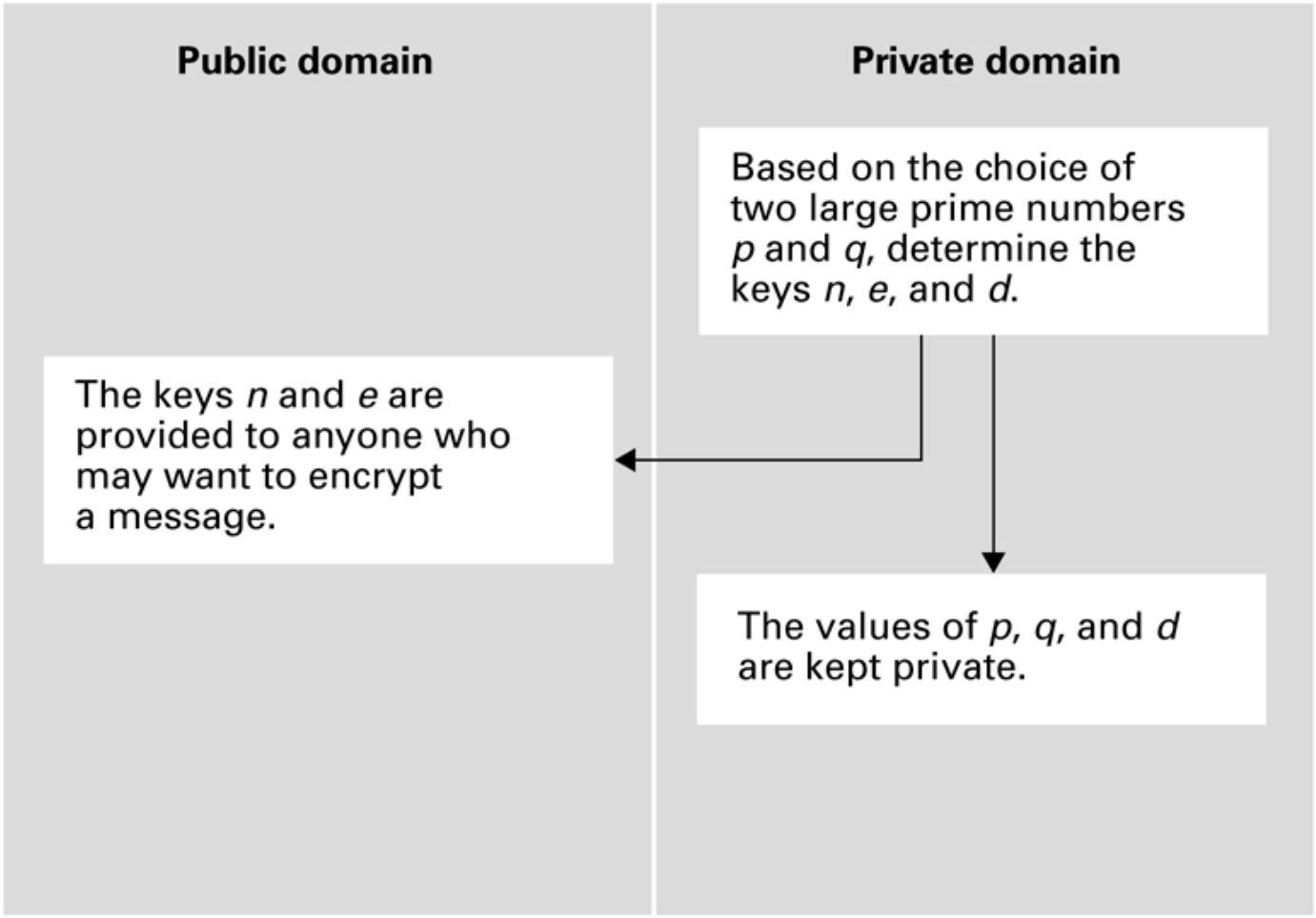
# Decrypting the Message 100

- Decrypting keys: d = 29, n = 91
- $100_{two} = 4_{ten}$
- $4^d = 4^{29} = 288{,}230{,}376{,}151{,}711{,}744$
- 288,230,376,151,711,744 ÷ 91 has a remainder of 23
- $23_{ten} = 10111_{two}$
- Therefore, decrypted version of 100 is 10111.

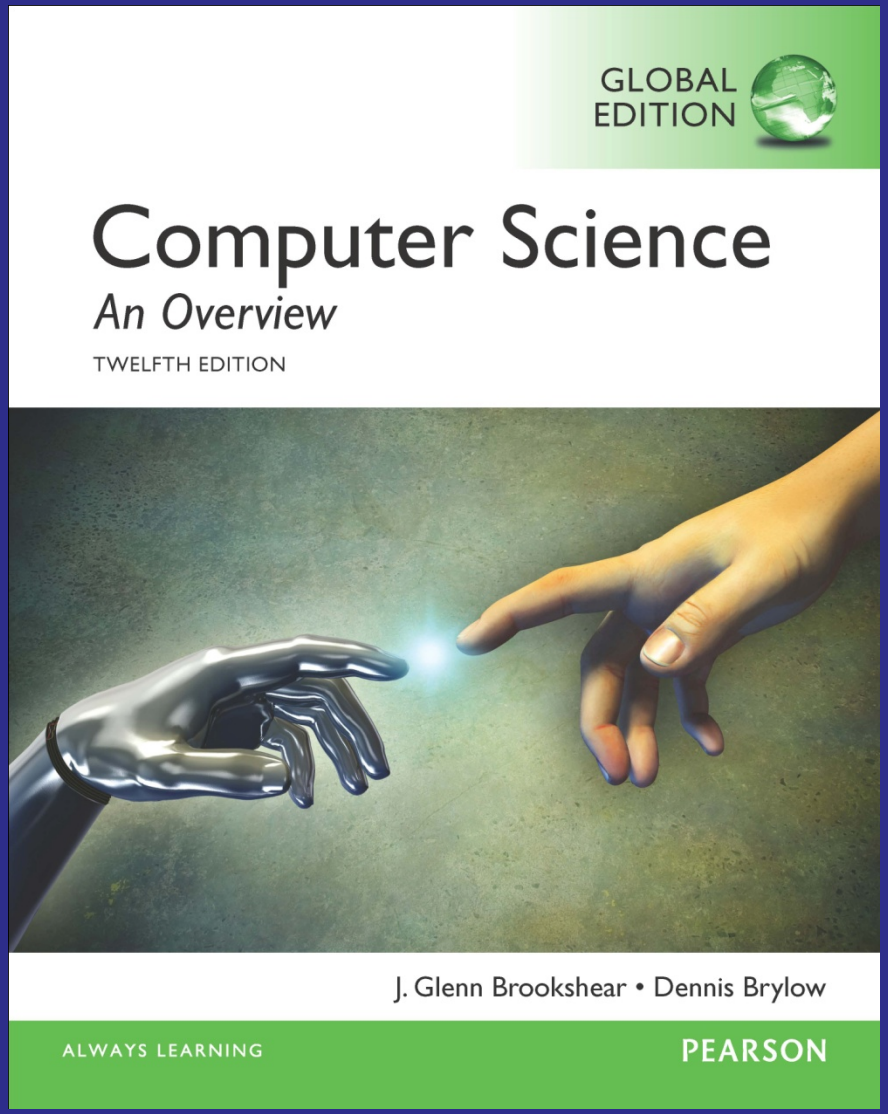
# Figure 12.13 **Public key cryptography**



# Figure 12.14 **Establishing an RSA public key encryption system**



# End of Chapter

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