ALIŞTIRMALAR

(Değişkenlerine Ayrılabilir Denklemler)

Variable Separable

2.1 $\cos^2 y\,dx+(1+e^{-x})\sin y\,dy=0$ ANS $\ln(e^x+1)=-\dfrac{1}{\cos y}+C;\ \cos y=0$

2.2 $\dfrac{dy}{dx}=\dfrac{x^3e^{x^2}}{y\ln y}$ ANS $y^2\left(\ln y-\frac{1}{2}\right)=e^{x^2}(x^2-1)+C$

2.3 $x\cos^2 y\,dx+e^x\tan y\,dy=0$ ANS $e^{-x}(x+1)=\dfrac{1}{2\cos^2 y}+C;\ \cos y=0$

2.4 $x\,(y^2+1)\,dx+(2y+1)\,e^{-x}\,dy=0$

ANS $(x-1)e^x+\ln(y^2+1)+\tan^{-1}y=C$

2.5 $xy^3\,dx+e^{x^2}\,dy=0$ ANS $e^{-x^2}+\dfrac{1}{y^2}=C;\ y=0$

2.6 $x\cos^2 y\,dx+\tan y\,dy=0$ ANS $x^2+\tan^2 y=C$

2.7 $xy^3\,dx+(y+1)e^{-x}\,dy=0$ ANS $e^x(x-1)-\dfrac{1}{y}-\dfrac{1}{2y^2}=C;\ y=0$

(Homojen Denklemler ve Özel Dönüşümler)

Homogeneous and Special Transformations

2.8 $\dfrac{dy}{dx}+\dfrac{x}{y}+2=0$ **ANS** $\ln|x+y|+\dfrac{x}{x+y}=C;\ y=-x$

2.9 $x\,dy-y\,dx=x\cot\left(\dfrac{y}{x}\right)dx$ **ANS** $\cos\left(\dfrac{y}{x}\right)=\dfrac{C}{x}$

2.10 $\left[x\cos^2\left(\dfrac{y}{x}\right)-y\right]dx+x\,dy=0$ **ANS** $\ln|x|+\tan\dfrac{y}{x}=C;\ \cos\dfrac{y}{x}=0$

2.11 $x\,dy=y(1+\ln y-\ln x)\,dx$ **ANS** $y=xe^{Cx}$

2.12 $xy\,dx+(x^2+y^2)\,dy=0$ **ANS** $y^2(2x^2+y^2)=C$

2.13 $\left[1+\exp\left(-\dfrac{y}{x}\right)\right]dy+\left(1-\dfrac{y}{x}\right)dx=0$ **ANS** $x\exp\left(\dfrac{y}{x}\right)+y=C$

2.14 $(x^2-xy+y^2)\,dx-xy\,dy=0$ **ANS** $(y-x)e^{y/x}=C$

2.15 $(3+2x+4y)y'=1+x+2y$

ANS $8y-4x+\ln|4x+8y+5|=C;\ 4x+8y+5=0$

2.16 $y'=\dfrac{2x+y-1}{x-y-2}$ **ANS** $\sqrt{2}\tan^{-1}\dfrac{y+1}{\sqrt{2}(x-1)}=\ln[(y+1)^2+2(x-1)^2]+C$

2.17 $(y+2)dx=(2x+y-4)dy$ **ANS** $(y+2)^2=C(x+y-1);\ y=1-x$

2.18 $y'=\sin^2(x-y)$ **ANS** $x=\tan(x-y)+C;\ x-y=\dfrac{\pi}{2}\pm k\pi,\ k=0,1,2,\ldots$

2.19 $\dfrac{dy}{dx}=(x+1)^2+(4y+1)^2+8xy+1$ **ANS** $\dfrac{2}{3}(x+4y+1)=\tan(6x+C)$

(Tam Diferansiyel Denklemler)

Exact Differential Equations

2.20 $(3x^2 + 6xy^2)\,dx + (6x^2y + 4y^3)\,dy = 0$ **Ans** $x^3 + 3x^2y^2 + y^4 = C$

2.21 $(2x^3 - xy^2 - 2y + 3)\,dx - (x^2y + 2x)\,dy = 0$

Ans $x^4 - x^2y^2 - 4xy + 6x = C$

2.22 $(xy^2 + x - 2y + 3)\,dx + x^2y\,dy = 2(x + y)\,dy$

Ans $x^2y^2 + x^2 + 6x - 4xy - 2y^2 = C$

2.23 $3y(x^2 - 1)\,dx + (x^3 + 8y - 3x)\,dy = 0$, when $x = 0,\ y = 1$

Ans $x^3y - 3xy + 4y^2 = 4$

2.24 $(x^2 + \ln y)\,dx + \dfrac{x}{y}\,dy = 0$ **Ans** $\frac{1}{3}x^3 + x \ln y = C$

2.25 $2x(3x + y - ye^{-x^2})\,dx + (x^2 + 3y^2 + e^{-x^2})\,dy = 0$

Ans $2x^3 + x^2y + ye^{-x^2} + y^3 = C$

2.26 $(3 + y + 2y^2 \sin^2 x)\,dx + (x + 2xy - y \sin 2x)\,dy = 0$

Ans $3x + xy + xy^2 - \frac{1}{2}y^2 \sin 2x = C$

2.27 $(2xy + y^2)\,dx + (x^2 + 2xy + y^2)\,dy = 0$ **Ans** $x^2y + xy^2 + \frac{1}{3}y^3 = C$

(İntegral Çarpanı)

Integrating Factors

2.28 $(x^2 - \sin^2 y)\,dx + x \sin 2y\,dy = 0$ ANS $x + \dfrac{\sin^2 y}{x} = C$

2.29 $y(2x - y + 2)\,dx + 2(x - y)\,dy = 0$ ANS $ye^x(2x - y) = C$

2.30 $(4xy + 3y^2 - x)\,dx + x(x + 2y)\,dy = 0$ ANS $4x^4y + 4x^3y^2 - x^4 = C$

2.31 $y\,dx + x(y^2 + \ln x)\,dy = 0$ ANS $3y \ln x + y^3 = C$

2.32 $(x^2 + 2x + y)\,dx + (3x^2y - x)\,dy = 0$ ANS $x + 2\ln|x| - \dfrac{y}{x} + \dfrac{3}{2}y^2 = C$

2.33 $y^2\,dx + (xy + y^2 - 1)\,dy = 0$ ANS $xy + \frac{1}{2}y^2 - \ln|y| = C$

2.34 $3(x^2 + y^2)\,dx + x(x^2 + 3y^2 + 6y)\,dy = 0$ ANS $xe^y(x^2 + 3y^2) = C$

2.35 $2y(x + y + 2)\,dx + (y^2 - x^2 - 4x - 1)\,dy = 0$

ANS $x^2 + 4x + 2xy + y^2 + 1 = Cy$

2.36 $(2 + y^2 + 2x)\,dx + 2y\,dy = 0$ ANS $e^x(2x + y^2) = C$

2.37 $(2xy^2 - y)\,dx + (y^2 + x + y)\,dy = 0$ ANS $x^2 - \dfrac{x}{y} + y + \ln|y| = C$

2.38 $y(x + y)\,dx + (x + 2y - 1)\,dy = 0$ ANS $e^x(xy + y^2 - y) = C$

2.39 $2x(x^2 - \sin y + 1)\,dx + (x^2 + 1)\cos y\,dy = 0$ ANS $\ln(x^2 + 1) + \dfrac{\sin y}{x^2 + 1} = C$

2.40 Consider a homogeneous differential equation of the form

$$M(u)\,dx + N(u)\,dy = 0, \qquad u = \frac{y}{x}.$$

If $Mx + Ny = 0$, i.e., $M(u) + N(u)u = 0$, show that $\dfrac{1}{xM}$ is an integrating factor.

Method of Inspection

2.41 $(x^2+y+y^2)\,dx - x\,dy = 0$ **ANS** $x - \tan^{-1}\dfrac{y}{x} = C$

2.42 $\left(x-\sqrt{x^2+y^2}\right)dx + \left(y-\sqrt{x^2+y^2}\right)dy = 0$ **ANS** $\sqrt{x^2+y^2} - x - y = C$

2.43 $y\sqrt{1+y^2}\,dx + \left(x\sqrt{1+y^2} - y\right)dy = 0$ **ANS** $xy - \sqrt{1+y^2} = C$

2.44 $y^2\,dx - (xy+x^3)\,dy = 0$ **ANS** $\dfrac{1}{2}\left(\dfrac{y}{x}\right)^2 + y = C$

2.45 $y\,dx - x\,dy - 2x^3\tan\dfrac{y}{x}\,dx = 0$ **ANS** $\sin\dfrac{y}{x} = Ce^{-x^2}$

2.46 $(2x^2y^2+y)\,dx + (x^3y-x)\,dy = 0$ **ANS** $x^2y + \ln\left|\dfrac{x}{y}\right| = C$

2.47 $y^2\,dx + \left[xy + \tan(xy)\right]dy = 0$ **ANS** $y\sin(xy) = C$

2.48 $(2x^2y^4-y)\,dx + (4x^3y^3-x)\,dy = 0$ **ANS** $2xy^2 + \dfrac{1}{xy} = C$

Integrating Factors by Groups

2.49 $(x^2y^3+y)\,dx + (x^3y^2-x)\,dy = 0$ **ANS** $\dfrac{1}{2}x^2y^2 + \ln\left|\dfrac{x}{y}\right| = C$

2.50 $y(y^2+1)\,dx + x(y^2-x+1)\,dy = 0$ **ANS** $\dfrac{1}{xy} - \dfrac{1}{y} - \tan^{-1}y = C$

2.51 $y^2\,dx + (e^x-y)\,dy = 0$ **ANS** $-ye^{-x} + \ln|y| = C$

2.52 $(x^2y^2-2y)\,dx + (x^3y-x)\,dy = 0$ **ANS** $\ln|xy| + \dfrac{1}{x^2y} = C$

2.53 $(2x^3y+y^3)\,dx - (x^4+2xy^2)\,dy = 0$ **ANS** $4x^{-\frac{1}{3}}y^{\frac{2}{3}} - x^{\frac{8}{3}}y^{-\frac{4}{3}} = C$

(Birinci Mertebeden Lineer (Doğrusal) Denklemler)

Linear First-Order Equations

2.54 $(1 + y\cos x)\,dx - \sin x\,dy = 0$ ANS $y = -\cos x + C\sin x$

2.55 $(\sin^2 y + x\cot y)\,y' = 1$ ANS $x = \sin y\,(C - \cos y)$

2.56 $dx - (y - 2xy)\,dy = 0$ ANS $2x = 1 + C\exp(-y^2)$

2.57 $dx - (1 + 2x\tan y)\,dy = 0$ ANS $2x\cos^2 y = y + \sin y\cos y + C$

2.58 $\dfrac{dy}{dx}\left(y^3 + \dfrac{x}{y}\right) = 1$ ANS $x = \frac{1}{3}y^4 + Cy$

2.59 $dx + (x - y^2)\,dy = 0$ ANS $x = y^2 - 2y + 2 + Ce^{-y}$

2.60 $y^2\,dx + (xy + y^2 - 1)\,dy = 0$ ANS $y^2 + 2xy - 2\ln|y| = C$

2.61 $y\,dx = (e^y + 2xy - 2x)\,dy$ ANS $y^2 x = Ce^{2y} - (y+1)e^y$

2.62 $(2x + 3)y' = y + (2x + 3)^{1/2},\ y(-1) = 0$ ANS $2y = \sqrt{2x+3}\,\ln|2x+3|$

2.63 $y\,dx + (y^2 e^y - x)\,dy = 0$ ANS $x = Cy - ye^y$

2.64 $y' = 1 + 3y\tan x$ ANS $y = \dfrac{1}{\cos^3 x}\left(\sin x - \frac{1}{3}\sin^3 x + C\right)$

2.65 $(1 + \cos x)\,y' = \sin x\,(\sin x + \sin x\cos x - y)$

ANS $y = (1 + \cos x)\,(x - \sin x + C)$

2.66 $y' = (\sin^2 x - y)\cos x$ ANS $y = \sin^2 x - 2\sin x + 2 + Ce^{-\sin x}$

2.67 $xy' - ny - x^{n+2}e^x = 0,\ \ n = \text{constant}$ ANS $y = x^n\left[e^x(x-1) + C\right]$

2.68 $(1+x)\dfrac{dy}{dx} - y = x(1+x)^2$ ANS $y = (1+x)\left(\frac{1}{2}x^2 + C\right)$

2.69 $(1+y)\,dx + \left[x - y(1+y)^2\right]dy = 0$ ANS $x = \dfrac{1}{1+y}\left(\dfrac{y^4}{4} + \dfrac{2y^3}{3} + \dfrac{y^2}{2} + C\right)$

2.70 Consider the first-order differential equation

$$\frac{dy}{dx} = \alpha(x)\,F(y) + \beta(x)\,G(y)$$

If $\dfrac{G'(y)F(y) - G(y)F'(y)}{F(y)} = a = \text{constant}$, then the transformation $u = \dfrac{G(y)}{F(y)}$ reduces the differential equation to a first-order linear differential equation. Show that the general solution of the differential equation is given by

$$\frac{G(y)}{F(y)} = \exp\left[a\int \beta(x)dx\right]\left\{a\int \alpha(x)\exp\left[-a\int \beta(x)dx\right]dx + C\right\}.$$

2.71 The *Riccati equation* is given by $y' = \alpha(x)y^2 + \beta(x)y + \gamma(x)$.

1. If one solution of this equation, say $y_1(x)$, is known, then the general solution can be found by using the transformation $y = y_1 + \dfrac{1}{u}$, where u is a new dependent variable. Show that u is given by

$$u = e^{-\int P(x)\,dx\,dx}\left[\int Q(x)\,e^{\int P(x)\,dx\,dx}\,dx + C\right],$$

   where $P(x) = 2\alpha(x)y_1(x) + \beta(x)$ and $Q(x) = -\alpha(x)$.

2. For the differential equation $y' + y^2 = 1 + x^2$, first guess a solution $y_1(x)$ and then use the result of Part 1 to find the general solution $y(x)$.

ANS $y = x + \dfrac{e^{-x^2}}{\int e^{-x^2}dx + C}$

(Bernoulli Diferansiyel Denklemi)

Bernoulli Differential Equations

2.72 $3xy' - 3xy^4 \ln x - y = 0$ **Ans** $\frac{1}{y^3} = -\frac{3}{4}x(2\ln x - 1) + \frac{C}{x}; \quad y = 0$

2.73 $\frac{dy}{dx} = \frac{4x^3y^2}{x^4y + 2}$ **Ans** $x^4 = -\frac{1}{y} + Cy; \quad y = 0$

2.74 $y(6y^2 - x - 1)\,dx + 2x\,dy = 0$ **Ans** $\frac{1}{y^2} = \frac{1}{x}(6 + Ce^{-x}); \quad y = 0$

2.75 $(1 + x)(y' + y^2) - y = 0$ **Ans** $\frac{1}{y} = \frac{1}{1+x}\left(\frac{x^2}{2} + x + C\right); \quad y = 0$

2.76 $xyy' + y^2 - \sin x = 0$ **Ans** $x^2y^2 = -2x\cos x + 2\sin x + C$

2.77 $(2x^3 - y^4)\,dx + xy^3\,dy = 0$ **Ans** $y^4 = 8x^3 + Cx^4$

2.78 $y' - y\tan x + y^2\cos x = 0$ **Ans** $\frac{1}{y} = \cos x\,(x + C); \quad y = 0$

2.79 $6y^2\,dx - x(2x^3 + y)\,dy = 0$ **Ans** $(y - 2x^3)^2 = Cyx^6; \quad y = 0$

(Bağımlı veya Bağımsız Değişkene Göre Çözülebilen Denklemler)

Equation Solvable for the Independent or Dependent Variable

2.80 $xy'^3 - yy'^2 + 1 = 0$ ANS $y = Cx + \frac{1}{C^2}$; $4y^3 = 27x^2$

2.81 $y = xy' + y'^3$ ANS $y = Cx + C^3$; $4x^3 + 27y^2 = 0$

2.82 $x(y'^2 - 1) = 2y'$ ANS $x = \frac{2p}{p^2-1}$, $y = \frac{2}{p^2-1} - \ln|p^2-1| + C$

2.83 $xy'(y' + 2) = y$ ANS $y = -x$; $y = \pm 2C\sqrt{x} + C^2$

2.84 $x = y'\sqrt{y'^2 + 1}$ ANS $x = p\sqrt{p^2+1}$, $3y = \sqrt{p^2+1}(2p^2-1) + C$

2.85 $2y'^2(y - xy') = 1$ ANS $y = Cx + \frac{1}{2C^2}$; $8y^3 = 27x^2$

2.86 $y = 2xy' + y^2y'^3$ ANS $y^2 = 2Cx + C^3$; $32x^3 + 27y^4 = 0$; $y = 0$

2.87 $y'^3 + y^2 = xyy'$ ANS $p^3 + y^2 = xyp$, $\frac{p^4}{2y^2} - p = C$; $y = 0$

2.88 $2xy' - y = y' \ln(yy')$ ANS $2x = 1 + 2\ln|y|$; $y^2 = 2Cx - C\ln C$

2.89 $y = xy' - x^2y'^3$ ANS $y = 0$; $xp^2 = C\sqrt{|p|} - 1$, $y = xp - x^2p^3$

2.90 $y(y - 2xy')^3 = y'^2$ ANS $27x^2y^2 = 1$; $y^2 = 2C^3x + C^2$

2.91 $y + xy' = 4\sqrt{y'}$ ANS $x = \frac{\ln p + C}{\sqrt{p}}$, $y = \sqrt{p}(4 - \ln p - C)$; $y = 0$

2.92 $2xy' - y = \ln y'$ ANS $x = \frac{1}{p} + \frac{C}{p^2}$, $y = 2\left(1 + \frac{C}{p}\right) - \ln p$

(Basit Yüksek Mertebeden Diferansiyel Denklemler)

Simple Higher-Order Differential Equations

2.93 $y'' = 2yy'^3$ **ANS** $y = C;\ \ 3x + y^3 + C_1 y = C_2$

2.94 $yy'' = y'^2 - y'^3$ **ANS** $y = C;\ \ C_1 \ln|y| + y = x + C_2$

2.95 $xy''' = (1-x)\,y''$ **ANS** $y = C_1(x+2)e^{-x} + C_2 x + C_3$

2.96 $y'' = e^x y'^2$ **ANS** $y = \dfrac{1}{C_1}\ln|1 + C_1 e^{-x}| + C_2;\ \ y = C$

2.97 $yy'' + y'^2 = 0$ **ANS** $y = C;\ \ \frac{1}{2}y^2 = C_1 x + C_2$

2.98 $1 + y'^2 = 2yy''$ **ANS** $4(C_1 y - 1) = C_1^2 (x + C_2)^2$

2.99 $xy'' = y'(\ln y' - \ln x)$

ANS $y = \dfrac{1}{C_1} e^{C_1 x + 1}\left(x - \dfrac{1}{C_1}\right) + C_2;\ \ y = \frac{1}{2}ex^2 + C$

2.100 $3yy'y'' - y'^3 + 1 = 0$ ANS $3(C_1y + 1)^{2/3} - 2C_1x = C_2;\ \ y = x$

2.101 $y'' - y'^2 - 1 = 0$ ANS $y = -\ln\left|\cos(x + C_1)\right| + C_2$

2.102 $x^3y'' - x^2y' = 3 - x^2$ ANS $y = \dfrac{1}{x} + x + C_1x^2 + C_2$

2.103 $2y'' = y'^3 \sin 2x,\ \ y(0) = 1,\ \ y'(0) = 1$ ANS $y = 1 + \ln\left|\sec x + \tan x\right|$

2.104 $x\dfrac{d^2y}{dx^2} = 2 - \dfrac{dy}{dx}$ ANS $y = 2x + C_1\ln|x| + C_2$

2.105 $y'' = 3\sqrt{y},\quad y(0) = 1,\ \ y'(0) = 2$ ANS $y = \left(\pm\frac{1}{2}x + 1\right)^4$

2.106 $x\dfrac{d^2y}{dx^2} = \dfrac{dy}{dx} + x\sin\left(\dfrac{1}{x}\cdot\dfrac{dy}{dx}\right)$

ANS $y = \left(x^2 + \dfrac{1}{C_1^2}\right)\tan^{-1}C_1x - \dfrac{x}{C_1} + C_2;\quad y = \dfrac{k\pi}{2}x^2 + C,\ k = 0, \pm1, \pm2, \ldots$

2.107 $yy'' = y'^2(1 - y'\sin y - yy'\cos y)$

ANS $y = C;\ \ x = -\cos y + C_1\ln|y| + C_2$

2.108 $y'' + xy' = x$ ANS $y = x + C_1\displaystyle\int e^{-\frac{1}{2}x^2}dx + C_2$

2.109 $xy'' - y'^3 - y' = 0$ ANS $x^2 + (y - C_1)^2 = C_2;\ \ y = C$

2.110 $y(1 - \ln y)y'' + (1 + \ln y)y'^2 = 0$

ANS $y = C;\ \ (C_1x + C_2)(\ln y - 1) + 1 = 0$

(2.111-2.160 Bu Bölüm için Genel Tekrar Problemleri)

Review Problems

2.111 $xy^2(xy' + y) = 1$ **Ans** $2x^3y^3 - 3x^2 = C$

2.112 $5y + y'^2 = x(x + y')$ **Ans** $4y = x^2;\ 5y = -5x^2 + 5Cx - C^2$

2.113 $y' = \dfrac{y+2}{x+1} + \tan\dfrac{y-2x}{x+1}$

Ans $\sin\dfrac{y-2x}{x+1} = C(x+1);\quad \dfrac{y+2}{x+1} = n\pi + 2,\ n = 0, \pm 1, \pm 2, \ldots$

2.114 $y''(e^x + 1) + y' = 0$ **Ans** $y = C_1(x - e^{-x}) + C_2$

2.115 $xy' = y - xe^{y/x}$ **Ans** $y = -x \ln\left|\ln|Cx|\right|$

2.116 $(1 + y^2 \sin 2x)\,dx - 2y\cos^2 x\,dy = 0$ **Ans** $x - y^2\cos^2 x = C$

2.117 $(2\sqrt{xy} - y)\,dx - x\,dy = 0,\ x > 0,\ y > 0$ **Ans** $\sqrt{xy} - x = C$

2.118 $y'' + y'^2 = 2e^{-y}$ **Ans** $e^y + C_1 = (x + C_2)^2$

2.119 $y' = e^{xy'/y}$ **Ans** $y = ex;\quad Cx = \ln|Cy|$

2.120 $(2x^3y^2 - y)\,dx + (2x^2y^3 - x)\,dy = 0$ **Ans** $x^2 + y^2 + \dfrac{1}{xy} = C$

2.121 $(y - 1 - xy)\,dx + x\,dy = 0$ **Ans** $xy + 1 = Ce^x$

2.122 $xy' - y = x\tan\dfrac{y}{x}$ **Ans** $\sin\dfrac{y}{x} = Cx$

2.123 $y' + \dfrac{y}{x} = e^{xy}$ **Ans** $-e^{-xy} = \frac{1}{2}x^2 + C$

2.124 $yy'' - yy' = (y')^2$ **Ans** $\ln\left|\ln|y| + C_2\right| = x + C_1;\quad y = C$

2.125 $2y\,dx - x\left[\ln(x^2y) - 1\right]dy = 0$ **Ans** $\ln(x^2y) - Cy = 0$

2.126 $y' = \dfrac{1}{xy + x^3y^3}$ **Ans** $\dfrac{1}{x^2} = 1 - y^2 + Ce^{-y^2}$

2.127 $y' = 2\left(\dfrac{y+2}{x+y-1}\right)^2$ **Ans** $y + 2 = C\exp\left[-2\tan^{-1}\dfrac{y+2}{x-3}\right]$

2.128 $(e^x + 3y^2)\,dx + 2xy\,dy = 0$ **Ans** $e^x(x^2 - 2x + 2) + x^3y^2 = C$

2.129 $(xy + 2x^3y)\,dx + x^2dy = 0$ **Ans** $xye^{x^2} = C$

2.130 $x(y')^2 - 2yy' + 4x = 0$ **Ans** $y = \dfrac{C}{2}x^2 + \dfrac{2}{C};\quad y = \pm 2x$

2.131 $y''' = 2(y''-1)\cot x$ ANS $y = (C_2+1)\dfrac{x^2}{2} + \dfrac{C_2}{4}\cos 2x + C_1 x + C_0$

2.132 $(y+3x^4y^2)\,dx + (x+2x^2y^3)\,dy = 0$ ANS $-\dfrac{1}{xy} + x^3 + y^2 = C;\quad y = 0$

2.133 $xy' = y + \sqrt{x^2-y^2},\ x>0,\ |y| \leqslant |x|$ ANS $\sin^{-1}\dfrac{y}{x} = \ln|x| + C;\ y = \pm x$

2.134 $2y(xe^{x^2} + y\sin x\cos x)\,dx + (2e^{x^2} + 3y\sin^2 x)\,dy = 0$

ANS $y^2e^{x^2} + y^3\sin^2 x = C$

2.135 $\cos y\,dx + \sin y(x - \sin y\cos y)\,dy = 0$

ANS $x = \cos y\left(\ln|\sec y + \tan y| - \sin y + C\right)$

2.136 $y^3\,dx + (3x^2 - 2xy^2)\,dy = 0$ ANS $y^3 = Ce^{y^2/x}$

2.137 $(y'+1)\ln\dfrac{y+x}{x+3} = \dfrac{y+x}{x+3}$ ANS $\ln\dfrac{y+x}{x+3} = 1 + \dfrac{C}{x+y}$

2.138 $2x^3yy' + 3x^2y^2 + 7 = 0$ ANS $x^3y^2 + 7x = C$

2.139 $\left(x - y\cos\dfrac{y}{x}\right)dx + x\cos\dfrac{y}{x}\,dy = 0$ ANS $\sin\dfrac{y}{x} = C - \ln|x|$

2.140 $x^2(x\,dy - y\,dx) = (x+y)y\,dx$ ANS $\ln\left|\dfrac{x}{y} + 1\right| = \dfrac{1}{x} + C$

2.141 $(y^4+xy)\,dx + (xy^3 - x^2)\,dy = 0$ ANS $2xy + \left(\dfrac{x}{y}\right)^2 = C;\quad y = 0$

2.142 $(x^2 + 3\ln y)\,dx - \dfrac{x}{y}\,dy = 0$ ANS $x^2 + \ln y = Cx^3$

2.143 $xy'' = y' + x$ ANS $4y = x^2\left(2\ln|x| + C_1\right) + C_2$

2.144 $y\,dx + (xy - x - y^3)\,dy = 0$ ANS $x = y(y-1) + Cye^{-y};\quad y = 0$

2.145 $y + 2y^3y' = (x + 4y\ln y)\,y'$ ANS $\dfrac{x}{y} + y^2 - 2(\ln y)^2 = C$

2.146 $y\ln x\,\ln y\,dx + dy = 0$ ANS $x\ln x - x + \ln|\ln y| = C$

2.147 $(2x\sqrt{x} + x^2 + y^2)\,dx + 2y\sqrt{x}\,dy = 0$ ANS $\ln(x^2+y^2) + 2\sqrt{x} = C$

2.148 $\left[2x + y\cos(xy)\right]dx + x\cos(xy)\,dy = 0$ ANS $x^2 + \sin(xy) = C$

2.149 $yy'' - y^2y' - y'^2 = 0$ ANS $y = C;\quad \dfrac{1}{C_1}\ln\left|\dfrac{y}{y+C_1}\right| = x + C_2$

2.150 $2y' + x = 4\sqrt{y}$ ANS $(2\sqrt{y} - x)\ln\left|C(2\sqrt{y} - x)\right| = x;\quad 2\sqrt{y} = x$

2.151 $2y'^3 - 3y'^2 + x = y$ **Ans** $y = x-1;\ \ 4(x+C)^3 = 27(y+C)^2$

2.152 $y' - 6x\,e^{x-y} - 1 = 0$ **Ans** $e^{y-x} = 3x^2 + C$

2.153 $(1+y^2)y'' + y'^3 + y' = 0$

Ans $-C_1 y - (1+C_1^2)\ln|y - C_1| = x + C_2;\ \ y = C$

2.154 $(y\sin x + \cos^2 x)\,dx - \cos x\,dy = 0$ **Ans** $-y\cos x + \dfrac{x}{2} + \dfrac{\sin 2x}{4} = C$

2.155 $y(6y^2 - x - 1)\,dx + 2x\,dy = 0$ **Ans** $\dfrac{x}{y^2} = 6 + Ce^{-x};\ \ y = 0$

2.156 $y'(x - \ln y') = 1$ **Ans** $x = \ln p + p^{-1},\ \ y = p - \ln p + C$

2.157 $(1+\cos x)y' + \sin x\,(\sin x + \sin x\cos x - y) = 0$

Ans $\frac{1}{2}x - \frac{1}{4}\sin 2x + \frac{1}{3}\sin^3 x + y\cos x + y = C$

2.158 $x\,dx + \sin^2\left(\dfrac{y}{x}\right)(y\,dx - x\,dy) = 0$ **Ans** $\dfrac{y}{2x} - \dfrac{1}{4}\sin\dfrac{2y}{x} = \ln|x| + C$

2.159 $(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$

Ans $x^2e^y + \dfrac{x^2}{y} + \dfrac{x}{y^3} = C$

2.160 $(xy^3 - 1)\,dx + x^2y^2\,dy = 0$ **Ans** $2x^3y^3 - 3x^2 = C$