

# MATEMATİKSEL FORMÜLLER TABLOSU

## 1. Trigonometrik Özdeşlikler

### Trigonometric Functions

$$\begin{aligned} 1. \quad & \sin^2 \theta + \cos^2 \theta = 1 \\ & \implies 1 + \tan^2 \theta = \sec^2 \theta \\ & \implies 1 + \cot^2 \theta = \csc^2 \theta \end{aligned}$$

$$2. \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$3. \quad \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$4. \quad \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\begin{aligned} 5. \quad \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta \end{aligned}$$

$$6. \quad \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$7. \quad \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$8. \quad \sin A \sin B = -\frac{1}{2} [\cos(A+B) - \cos(A-B)]$$

$$9. \quad \cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$10. \quad \sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$11. \quad \cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

### Hyperbolic Functions

$$\begin{aligned} \cosh^2 x - \sinh^2 x &= 1 \\ \implies 1 - \tanh^2 x &= \operatorname{sech}^2 x \\ \implies \coth^2 x - 1 &= \operatorname{csch}^2 x \end{aligned}$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\begin{aligned} \cosh 2x &= \cosh^2 x + \sinh^2 x \\ &= 2 \cosh^2 x - 1 \\ &= 1 + 2 \sinh^2 x \end{aligned}$$

$$12. \sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$13. \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$14. \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$15. \cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$16. \text{Euler's Formula: } e^{i\theta} = \cos \theta + i \sin \theta$$

$$17. z^n = a(\cos \theta + i \sin \theta)$$

$$\implies z = \sqrt[n]{a} \left( \cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n} \right), \quad k = 0, 1, \dots, n-1$$

## 2. Türev

$$1. \frac{d}{dx} x^n = nx^{n-1}$$

$$2. \frac{d}{dx} e^x = e^x \implies \frac{d}{dx} a^x = a^x \ln a, \quad \because a^x = e^{x \ln a}$$

$$3. \frac{d}{dx} \ln x = \frac{1}{x} \implies \frac{d}{dx} \log_a x = \frac{1}{x \ln a}, \quad \because \log_a x = \frac{\ln x}{\ln a}$$

### Trigonometric Functions

$$4. \frac{d}{dx} \sin x = \cos x$$

$$5. \frac{d}{dx} \cos x = -\sin x$$

$$6. \frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = \sec^2 x$$

$$7. \frac{d}{dx} \cot x = -\frac{1}{\sin^2 x} = -\csc^2 x$$

$$8. \frac{d}{dx} \sec x = \frac{\sin x}{\cos^2 x} = \tan x \sec x$$

$$9. \frac{d}{dx} \csc x = -\frac{\cos x}{\sin^2 x} \\ = -\cot x \csc x$$

### Hyperbolic Functions

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \tanh x = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \coth x = -\frac{1}{\sinh^2 x} = -\operatorname{csch}^2 x$$

$$\frac{d}{dx} \operatorname{sech} x = -\frac{\sinh x}{\cosh^2 x} \\ = -\tanh x \operatorname{sech} x$$

$$\frac{d}{dx} \operatorname{csch} x = -\frac{\cosh x}{\sinh^2 x} \\ = -\coth x \operatorname{csch} x$$

### Inverse Trigonometric Functions

$$10. \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$11. \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$12. \frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx} \csc^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$$

### 3. Integral

$$1. \int x^n dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$2. \int \frac{1}{x} dx = \ln|x|, \quad x \neq 0 \qquad \int \ln x dx = x(\ln x - 1), \quad x > 0$$

$$3. \int e^{ax} dx = \frac{1}{a}e^{ax} \qquad \int b^{ax} dx = \frac{b^{ax}}{a \ln b}, \quad b > 0$$

#### Trigonometric Functions

$$4. \int \sin x dx = -\cos x$$

$$5. \int \cos x dx = \sin x$$

$$6. \int \tan x dx = -\ln|\cos x|$$

$$7. \int \cot x dx = \ln|\sin x|$$

$$8. \int \sec x dx = \ln|\sec x + \tan x| \\ = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| = \ln\left|\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right|$$

$$9. \int \csc x dx = \ln|\csc x - \cot x| \\ = \ln\left|\tan\frac{x}{2}\right|$$

$$10. \int \sin^2 x dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$

$$11. \int \cos^2 x dx = \frac{x}{2} + \frac{1}{4}\sin 2x$$

$$12. \int \tan^2 x dx = \tan x - x \qquad \int \tanh^2 x dx = -\tanh x + \frac{1}{2} \ln\left|\frac{\tanh x + 1}{\tanh x - 1}\right|$$

$$13. \int \cot^2 x dx = -\cot x - x \qquad \int \coth^2 x dx = -\coth x + \frac{1}{2} \ln\left|\frac{\coth x + 1}{\coth x - 1}\right|$$

$$14. \int \sec^2 x dx = \tan x \qquad \int \operatorname{sech}^2 x dx = \tanh x$$

$$15. \int \csc^2 x dx = -\cot x \qquad \int \operatorname{csch}^2 x dx = -\coth x$$

#### Hyperbolic Functions

$$\int \sinh x dx = \cosh x$$

$$\int \cosh x dx = \sinh x$$

$$\int \tanh x dx = \ln \cosh x$$

$$\int \coth x dx = \ln|\sinh x|$$

$$\int \operatorname{sech} x dx = \tan^{-1}(\sinh x)$$

$$\int \operatorname{csch} x dx = \ln\left|\tanh\frac{x}{2}\right|$$

$$\int \sinh^2 x dx = \frac{1}{4}\sinh 2x - \frac{x}{2}$$

$$\int \cosh^2 x dx = \frac{1}{4}\sinh 2x + \frac{x}{2}$$

16.  $\int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax$
17.  $\int x^2 \sin ax \, dx = \frac{2x}{a^2} \sin ax - \frac{a^2 x^2 - 2}{a^3} \cos ax$
18.  $\int x^3 \sin ax \, dx = \frac{3a^2 x^2 - 6}{a^4} \sin ax - \frac{a^2 x^3 - 6x}{a^3} \cos ax$
19.  $\int x^4 \sin ax \, dx = \frac{4a^2 x^3 - 24x}{a^4} \sin ax - \frac{a^4 x^4 - 12a^2 x^2 + 24}{a^5} \cos ax$
20.  $\int x \cos ax \, dx = \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax$
21.  $\int x^2 \cos ax \, dx = \frac{a^2 x^2 - 2}{a^3} \sin ax + \frac{2x}{a^2} \cos ax$
22.  $\int x^3 \cos ax \, dx = \frac{a^2 x^3 - 6x}{a^3} \sin ax + \frac{3a^2 x^2 - 6}{a^4} \cos ax$
23.  $\int x^4 \cos ax \, dx = \frac{a^4 x^4 - 12a^2 x^2 + 24}{a^5} \sin ax + \frac{4a^2 x^3 - 24x}{a^4} \cos ax$
24.  $\int e^{bx} \sin ax \, dx = e^{bx} \frac{b \sin ax - a \cos ax}{a^2 + b^2}$
25.  $\int e^{bx} \cos ax \, dx = e^{bx} \frac{a \sin ax + b \cos ax}{a^2 + b^2}$
26.  $\int x e^{ax} \, dx = \frac{ax - 1}{a^2} e^{ax}$
27.  $\int x^2 e^{ax} \, dx = \frac{a^2 x^2 - 2ax + 2}{a^3} e^{ax}$
28.  $\int x^3 e^{ax} \, dx = \frac{a^3 x^3 - 3a^2 x^2 + 6ax - 6}{a^4} e^{ax}$
29.  $\int x^4 e^{ax} \, dx = \frac{a^4 x^4 - 4a^3 x^3 + 12a^2 x^2 - 24ax + 24}{a^5} e^{ax}$
30.  $\int x^5 e^{ax} \, dx = \frac{a^5 x^5 - 5a^4 x^4 + 20a^3 x^3 - 60a^2 x^2 + 120ax - 120}{a^6} e^{ax}$
31.  $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a > 0$
32.  $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln \frac{a+x}{a-x} \quad \text{or} \quad \frac{1}{a} \tanh^{-1} \frac{x}{a}, \quad |x| < |a|$

$$33. \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \frac{x-a}{x+a} \quad \text{or} \quad -\frac{1}{a} \coth^{-1} \frac{x}{a}, \quad |x| > |a|$$

$$34. \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \quad \text{or} \quad -\cos^{-1} \frac{x}{a}$$

$$35. \int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1} \frac{x}{a} \quad \text{or} \quad \ln|x + \sqrt{x^2 - a^2}|$$

$$36. \int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1} \frac{x}{a} \quad \text{or} \quad \ln(x + \sqrt{x^2 + a^2})$$

$$37. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$38. \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

$$\text{or} \quad \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2})$$

$$39. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$$

$$\text{or} \quad \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}|$$

$$40. \int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \cosh^{-1} \frac{a}{x} \quad \text{or} \quad -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$41. \int \frac{1}{x\sqrt{x^2 + a^2}} dx = -\frac{1}{a} \sinh^{-1} \frac{a}{x} \quad \text{or} \quad -\frac{1}{a} \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$$

$$42. \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \cos^{-1} \frac{a}{x} \quad \text{or} \quad \frac{1}{a} \sec^{-1} \frac{x}{a}$$

$$43. \int \frac{1}{\sqrt{2ax - x^2}} dx = \cos^{-1} \left( 1 - \frac{x}{a} \right) \quad \text{or} \quad \sin^{-1} \left( \frac{x}{a} - 1 \right)$$

$$44. \int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} - 1 \right)$$

$$45. \int_0^{\frac{\pi}{2}} \begin{cases} \sin^n \theta \\ \cos^n \theta \end{cases} d\theta = \frac{(n-1)!!}{n!!} \times \begin{cases} \frac{1}{2}\pi, & \text{if } n \text{ is an even integer} \\ 1, & \text{if } n \text{ is an odd integer} \end{cases}$$

$$46. \int_0^{\frac{\pi}{2}} \sin^n \theta \cos^m \theta d\theta = \frac{(n-1)!! (m-1)!!}{(n+m)!!} \times \begin{cases} \frac{1}{2}\pi, & n, m \text{ even integers} \\ 1, & \text{otherwise} \end{cases}$$

$$\text{Def } n!! = \begin{cases} n \cdot (n-2) \cdots 5 \cdot 3 \cdot 1, & n > 0 \text{ odd integer} \\ n \cdot (n-2) \cdots 6 \cdot 4 \cdot 2, & n > 0 \text{ even integer} \\ 1, & n = 0 \end{cases}$$