INVERSE MODEL TO ESTIMATE O-D MATRIX FROM LINK TRAFFIC COUNTS USING ANT COLONY OPTIMIZATION

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Abstract- *Estimation of Origin–Destination (O-D) trip table has long been carried out with maximum entropy, generalized least squares, bi-level programming methods etc. All optimization models developed so far are either calculus based and mathematically lengthy. All approaches are prone to local optima. Therefore, there is a need for a new method which estimates O-D trip table and solves the traffic assignment models simultaneously. The main objective of this study is therefore to develop a model to estimate O-D trip table from link traffic counts using ant colony optimization (ACO) method based on inverse modelling technique. The ACO has been recently developed, as a population based meta-heuristic that has been successfully applied to several NP-hard combinatorial optimization problems. The core of ant's behavior is the communication between the ants by means of chemical pheromone trails, which enables them to find shortest paths between their nest and food sources. Ant Colony optimization O-D Estimation (ACODE) model is formulated as simultaneous optimization problem, the O-D trip matrices and stochastic user equilibrium (SUE) path and link flows are obtained simultaneously. The ACODE model is applied to an example transportation network which has 13 nodes with 19 links, 25 routes and 4 O-D pairs. O-D trip table is estimated using proposed ACODE model from given link traffic volumes. Results showed that inversely applied ACODE model for O-D matrix estimation from link traffic counts may estimate the O-D trips under SUE assumption.*

Keywords-O-D estimation, link traffic counts, ant colony optimization, stochastic user equilibrium

INTRODUCTION

The Origin–Destination (O–D) trip table estimation is an essential element of network based traffic models. Estimating O-D matrix from traffic counts on road links is of considerable importance. The O-D is also an essential ingredient in a wide variety of travel analysis and planning studies [1]. Over the past several decades, a considerable number of methods for O-D estimation have been reported in the literature. O-D matrix is the basic data for the traffic planning and management. It is a demand for traffic that flows from origins to destinations, which is expressed as a matrix to explore the movement of space flow. Statistical techniques have become popular in the estimation or updating of O-D matrix from traffic counts. The traditional way of estimating O-D from home-interview survey data is expensive [2]. Hence, generally, the estimates are based on small sample of home-interview data and thus the accuracy of the estimates suffers. This led the researchers to estimate the O-D from a variety of other data sources among which O-D estimation from link traffic counts has attracted lot of interest as the required data collection is simple and routine. The O-D demand matrix estimation methods in the literature and its advantages and disadvantages are given in the next section.

This paper is structured as follows. The next section reviews the O-D estimation matrix estimation methods. An improved ant colony optimization method and its solution procedure are proposed in Section 3. The algorithm defined to estimate the O-D matrix using improved ant colony optimization with inverse model from the link traffic counts is given in Section 4. In Section 5, a numerical example is carried out to present effectiveness for proposed algorithm. Finally, our conclusions can be seen in the last section.

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LITERATURE REVIEW

O-D matrix estimation has been studied by a few researchers and notable developments have been achieved at this concept of transportation network design. Reference [3]-[5] were interested in finding probable O-D movements in terms of link flows by deterministic assignment equation. Reference [6] was the first to attempt the equilibrium assignment based O-D estimation. The formulation, however, does not ensure a unique O-D solution because the formulation is not strictly convex in the O-D variables. Uniqueness of the solution is ensured if a target O-D is used [7]-[9]. These models require a complete set of link counts, a target O-D matrix and that the observed flows be in equilibrium.

Two significant methods which do not use entropy formulation and include generalized least squares estimation were developed by [10] and [11]. Reference [2] developed another method which also uses generalized least squares and also allows the explicit use of data describing the structure of the O-D. Reference [12] proposes a model which inferences about an O-D matrix from a single observation on a set of link flows. Two problems are discussed in this study; the first, the problem of reconstructing the actual number of O-D trips, and the second, estimation of mean O-D trip rates. A fast constrained recursive identification (CRI) algorithm is proposed to estimate O-D matrices by [13]. The basic idea of the CRI algorithm is to estimate intersection O-D matrices based on equality-constrained optimization. A Fuzzy inference based assignment algorithm to estimate O-D matrices from link volume counts is proposed by [1].

Reference [14] proposes a new model which has been formulated by using a new approach called the calibration and demand adjustment model (CDAM) based on bi-level programming which simultaneously estimates an O–D matrix and the parameters for the nested logit model. The algorithm iterates between the network equilibrium problem and that which is used to obtain a set of paths when equilibrium is attained, and the CDAM is restricted to the set of previously generated columns. The computational tests on the algorithm have been carried out using data from a multi-modal network in Madrid.

In this study, an improved ant colony optimization (IACO) based algorithm which is called ACODE is proposed to estimate O-D demands on transportation networks. The ACODE model considers Stochastic User Equilibrium (SUE) conditions for modeling drivers' route choice perceptions. The methodology is given in the next sections and the model is applied to a test network.

ANT COLONY OPTIMIZATION

Ant Colony Optimization (ACO) belongs to the class of biologically inspired heuristics. The ACO is the one of the most recent techniques for approximate optimization methods, was initiated by [15]. The core of ant's behavior is the communication between the ants by means of chemical *pheromone* trails, which enables them to find shortest paths between their nest and food sources. The Improved algorithm for ACO (IACO) that is proposed in this study is based on each ant searches only around the best solution of the previous iteration with coefficient β. It is very important for improving IACO's solution performance. IACO differs from other ACOs in that its feasible search space (FSS) is reduced with coefficient β and its best solution obtained using information on the previous iteration. At the core of IACO, ants search randomly the solution within the FSS to reach optimum or near-optimum values. At the end of the each iteration, only one of the ants is near to global minimum. After the first iteration, when global minimum is searched around the best solution of the previous iteration using β, the IACO will quickly reach to the global minimum. IACO is performed by modifying the algorithm proposed by [16]. The algorithm can be defined in the following way.

At the beginning of the first iteration, all ants search randomly best solution of the problem within the FSS. At the end of the first iteration, FSS is reduced by β and best solution obtained of the previous iteration is kept. Optimum solution is then searched in the reduced search space during the steps of algorithm progress. IACO reaches to the global minimum as ants find their routes in the limited space. β guides the bounds of search space throughout the IACO application. The main idea of proposed algorithm is given in Figure 1. Main advantageous of IACO is that the FSS is reduced with coefficient β and it uses the information taken from previous iteration. For example, consider a problem of five ants represents the formulation of the problem. Five ants being associated five random initial vectors. Only one of the solutions which were obtained at the end of the first iteration is near to global minimum. After the first iteration, FSS is reduced according to coefficient β and best solution (Ant 1 is the best solution, as shown in Fig. 1(a)) of the previous iteration. FSS is getting smaller during iteration progress as shown in Fig. 1(b). Coefficient β has been chosen according to the size of search space and constraints the problem in order not being trapped in bad local minimum. In IACO, let number of *m* ants being associated with *m* random initial vectors $(x^k \quad (k = 1,2,3,......m))$. Quantity of pheromone (τ_t) only intensifies around the best objective function value. The solution vector of the each ant is updated using (1).

$$x_t^{k(new)} = x_t^{k(old)} \pm \alpha \qquad (t = 1,2,....., I) \tag{1}$$

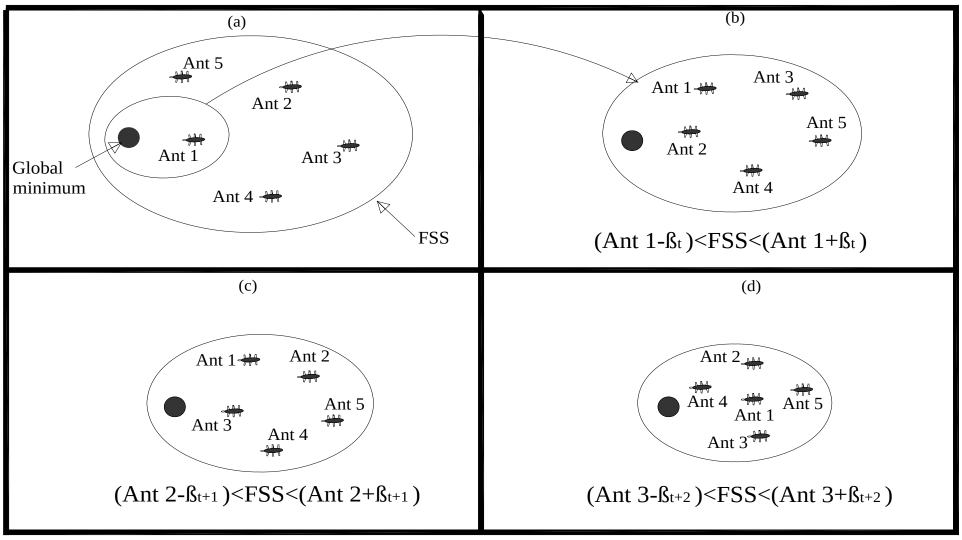

FIGURE 1
Main idea of IACO model (FSS is getting smaller during the optimization process using previous information and coefficient β)

where $x_t^{k(new)}$ is the solution vector of the k^{th} ant at iteration *t* (see Figure 2), $x_t^{k(old)}$ is the solution obtained from the previous step at iteration *t*, and α is a vector generated randomly to determine the length of jump.

Ant vector $x_t^{k(new)}$ obtained at t^{th} iteration in (1) is determined using the value of same ant obtained from previous step. At the last step of the each iteration, a new ant colony (see Figure 2, second loop) is developed as the number of colony size that is generated at the beginning of the each iteration. Steps of algorithm are illustrated in Figure 2. Quantity of pheromone (τ_t) is reduced to simulate the evaporation process of real ant colonies using (2). After reducing the number of pheromone, it is updated using (3). This process is repeated until the given number of iteration, *I*, is completed.

$$\tau_t = 0.1 * \tau_{t-1} \tag{2}$$

$$\tau_t = \tau_{t-1} + 0.01 * f(x_{t-1}^{best}) \tag{3}$$

In Equation (1), (+) sign is used when point x_t^k is on the left of global minimum on the *x* coordinate axis. (-) sign is used when point x_t^k is on the right of global minimum on the same axis. The direction of movement is defined by (4).

$$\bar{x}_t^{best} = x_t^{best} + (x_t^{best} * 0.01) \tag{4}$$

If $f(\overline{x}_t^{best}) \leq f(x_t^{best})$, (+) sign is used in (1). Otherwise, (-) sign is used. $(\pm)$ sign defines the search direction of movement to reach to the global minimum. α value is used to define the length of jump, and it will be gradually decreased in order not to pass over global minimum, as shown in Figure 2.

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Initialization
  FOR i=1 TO I          (I=iteration number)
      IF I=1 THEN generate m random ants within FSS
      ELSE reduce FSS with range [ x_{t-1}^{best} + β; x_{t-1}^{best} − β ]
      END IF
   FOR i=1 TO m
      Determine f(x_t^{best})        {old ant colony}
      Save x_t^{best}
   END
Pheromone update
      Pheromone evaporation using (2)
      Update pheromone trail using (3)
Solution phase
      Determine search direction using (4)
      Generate the values of α vector
   FOR i=1 TO m
    Determine the members of new colony using (1)   {new ant colony}
      Determine new f(x_t^{best})
       Save x_t^{best}
    END
      IF f(x_t^{best})^{new} ≤ f(x_t^{best})^{old} THEN x^{global min} = (x_t^{best})^{new}
      ELSE x^{global min} = (x_t^{best})^{old}
      END IF
        α_t = α_{t-1} * 0.99
      β_t = β_{t-1} * 0.99
    END
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FIGURE 2
Steps of IACO

The IACO is performed by means of reduced search space with β and using the information provided by previous solution. Moreover, IACO differs in terms of the generated new colony. Means that at the last step of the each iteration, a new colony is developed as a number of colony size, that is generated at the beginning of each iteration and the length of jump is applied to the same ant of the previous step at the iteration t to generate a new colony instead of applying the best ant, which obtained from previous iteration.

PROPOSED O-D MATRIX ESTIMATION TECHNIQUE

ACODE is SUE based method that considers perceived route travel times of drivers by using C-Logit route choice model [17]. SUE assignment can be expressed as a fixed-point problem in the link flow space, over the non-empty, compact and convex set of feasible link flow patterns by following the works by [18]-[19]. The fixed problem can be written as

$$v_a = \sum_{w \in W} \sum_{r \in R_w} q_w \delta_{ar}^w p_r^w \left(\mathbf{t^w} \right), a \in L \tag{5}$$

where v_a is the link traffic volume on link a, $a \in L$, q_w is the travel demand between the O-D pair $w \in W$, δ_{ar}^w is the link-path incidence matrix where δ_{ar}^w =1 if route r between O-D pair w uses link a and 0 otherwise, $p_r^w(\mathbf{t^w})$ is the probability of drivers choosing route $r \in R$ and $\mathbf{t^w}$ is a vector of travel times of all routes between the O-D pair w.

C-Logit model is adopted for the numerical calculation section of this study since application is quite easy. The probability of choosing route r may be expressed with C-Logit model is given in (6).

$$p_r^w \left(\mathbf{t^w} \right) = \frac{\exp\left(-\theta_0 t_r^w - \theta_1 CF_r^w \right)}{\sum_{k \in R_w} \exp\left(-\theta_0 t_k^w - \theta_1 CF_k^w \right)}, r \in R_w, w \in W \tag{6}$$

where CF_r^w is the commonality factor for route $r \in R_w$ and it represents the degree of similarity of route r with the other routes in the set of R_w between the O-D pair $w \in W$ and $\theta_0 - \theta_1$ are the parameter of the Gumble random variable, directly proportional to the standard error of perceived path costs. Several ways are suggested to specify the commonality factor that gives similar results with each other by Cascetta *et al*. (1996). The commonality factor used in this study is given in (7) as

$$CF_r^w = \sum_{a \in A} \delta_{ar}^w w_{ar}^w \ln N_a^w, r \in R_w, w \in W \tag{7}$$

where N_a^w is the number of routes, connecting O-D pair $w \in W$ that share link a and w_{ar}^w is the proportional weight of link a for route $r \in R_w$, specified as the fraction of total route travel time which is attributed to link a:

$$w_{ar}^w = \frac{t_a}{t_r^w} \tag{8}$$

where t_a is the link travel time and t_r^w is the corresponding route travel time.

SUE assignment part ends after finding route choice probabilities and new link flows (v_a^*) can be obtained by using (5) with respect to randomly generated O-D trips. Minimization of the following objective function provides requested O-D demands.

$$\min F(v_a, v_a^*) = \sum_{a \in L} \left(v_a - v_a^* \right)^2 \tag{9}$$

where, v_a is observed and v_a^* is obtained link flows from estimated O-D movements. A flowchart for the ACODE model is given in Figure 3.

APPLICATION OF PROPOSED METHOD TO ESTIMATE O-D TRIP TABLE

Let us now consider the application of the method described in the previous section. By way of illustration, consider the figure of network given in Figure 4, where the equilibrium path flows are generated randomly. This network is also studied by [20] in order to estimate origin destination matrices in congested traffic networks using column generation algorithm.

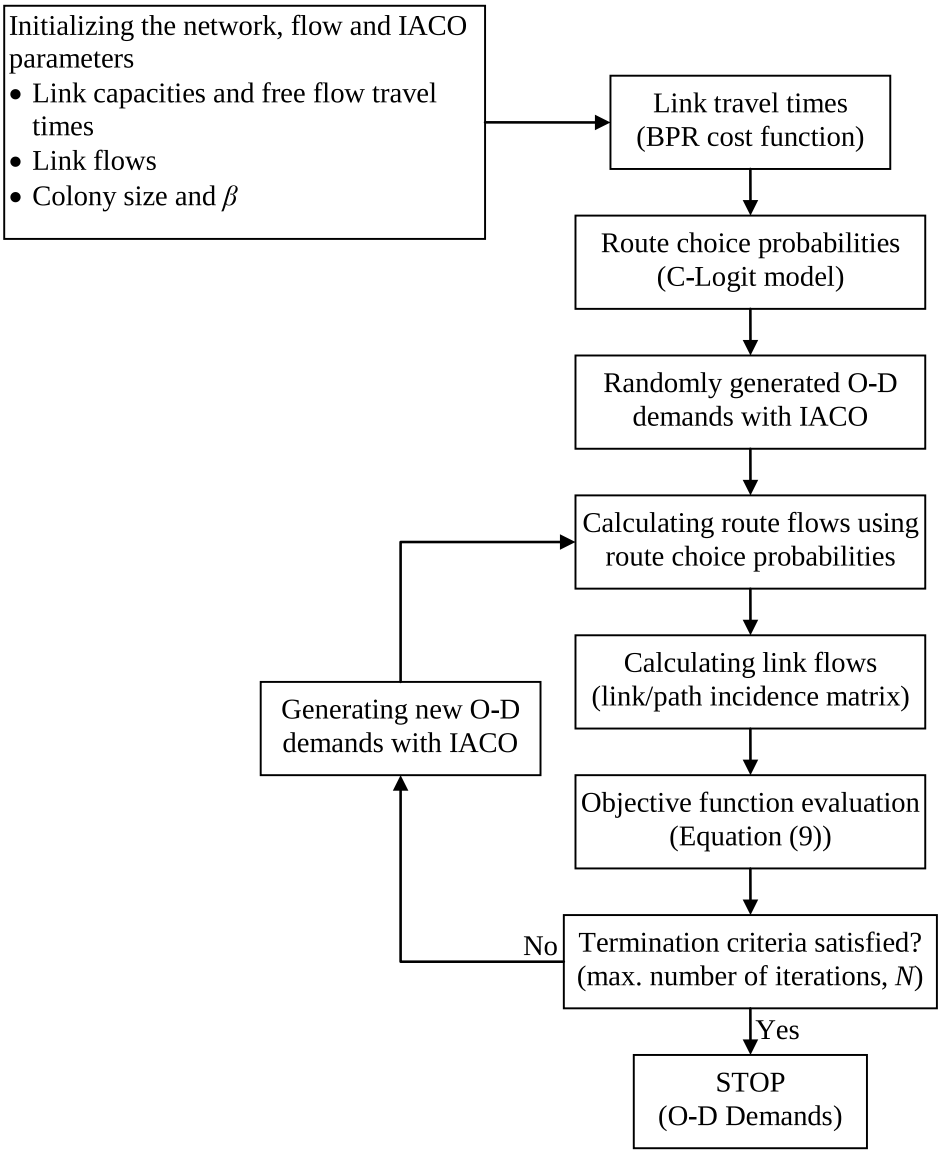

FIGURE 3
The solution algorithm of ACODE model

The standard Bureau of Public Roads (BPR) function which is given in (10) adopted with b_a=1 ve n_a=2, $\forall a \in L$ in order to find link costs. The other parameters of the BPR function are given in Table 2.

$$T_a(V_a) = t_a^0 \left[1 + b_a \left(\frac{V_a}{C_a}\right)^{n_a} \right] \tag{10}$$

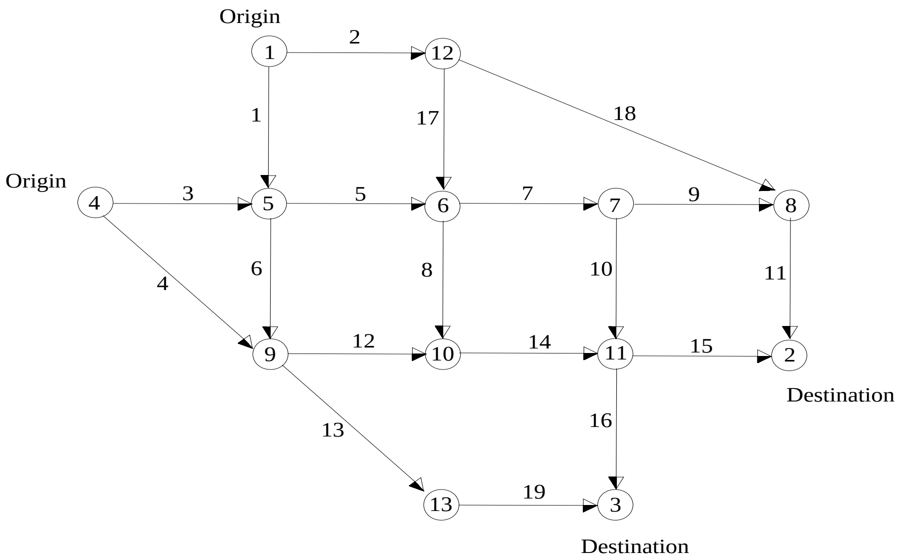

FIGURE 4
Test network ([21])

As shown in Figure 4, test network has 13 nodes with 19 links and 25 routes between 4 O-D pairs which are given in Table 1.

TABLE 1
Test network topology and equilibrium path flows

O-D movement	O-D pair	Path	Link sequence	Path flows	O-D movement	O-D pair	Path	Link sequence	Path flows
1	(1,2)	1	2-18-11	276	3	(1,3)	14	1-6-13-19	230
		2	1-5-7-9-11	112			15	1-5-7-10-16	107
		3	1-5-7-10-15	83			16	1-5-8-14-16	90
		4	1-5-8-14-15	76			17	1-6-12-14-16	105
		5	1-6-12-14-15	98			18	2-17-7-10-16	146
		6	2-17-7-9-11	145			19	2-17-8-14-16	123
		7	2-17-7-10-15	113	4	(4,3)	20	4-13-19	295
		8	2-17-8-14-15	98			21	4-12-14-16	157
2	(4,2)	9	4-12-14-15	386			22	3-6-13-19	201
		10	3-5-7-9-11	378			23	3-5-7-10-16	133
		11	3-5-7-10-15	261			24	3-5-8-14-16	105
		12	3-5-8-14-15	222			25	3-6-12-14-16	109
		13	3-6-12-14-15	254					

TABLE 2
Link cost parameters

Link	Free flow travel time, t_a^0	Link capacity, C_a
1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 19	10	2000
4	20	2000
13	20	2000
18	30	2000

In order to show the effectiveness of the ACODE, the evaluation of objective function during the iteration process is given in Figure 5. As can be seen in Figure, ACODE achieved to exact solution for given objective function about after 500 iterations.

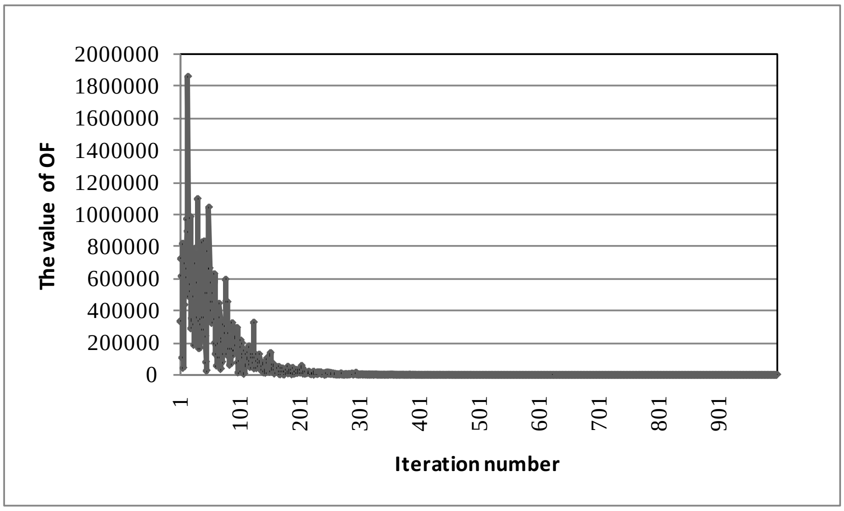

FIGURE 5
The evaluation of objective function (OF)

O-D demand matrix is obtained using ACODE model after 1000 iterations and given in Table 3.

TABLE 3
Obtained O-D trip table using ACODE model

Origin	Destination	
	2	3
1	1000	800
4	1500	1000

CONCLUSIONS

This paper deals with the development of a new algorithm to estimate O–D matrices from observed link traffic counts. A new approach called ACODE has been formulated based on SUE conditions. We have proved the existence of solutions for ACODE. Test network which has 13 nodes, 19 links, 25 routes and 4 O-D pairs is used to show the effectiveness of ACODE algorithm. Numerical results showed that the proposed ACODE algorithm is able to estimate O-D trip table from link traffic counts. Further, the results show that the proposed approach holds promise for successful application to large networks with complex flow patterns.

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