

ALİŞTIRMALAR

Aşağıdaki homojen (ikinci yansız) adi diferansiyel denklemlerin genel çözümlerini bulunuz.

4.1 $(D^3 - 2D^2 + D - 2)y = 0$ **ANS** $y_C = A \cos x + B \sin x + Ce^{2x}$

4.2 $(D^3 + D^2 + 9D + 9)y = 0$ **ANS** $y_C = A \cos 3x + B \sin 3x + Ce^{-x}$

4.3 $(D^3 + D^2 - D - 1)y = 0$ **ANS** $y_C = (C_0 + C_1x)e^{-x} + Ce^x$

4.4 $(D^3 + 8)y = 0$ **ANS** $y_C = e^x[A \cos(\sqrt{3}x) + B \sin(\sqrt{3}x)] + Ce^{-2x}$

4.5 $(D^3 - 8)y = 0$ **ANS** $y_C = e^{-x}[A \cos(\sqrt{3}x) + B \sin(\sqrt{3}x)] + Ce^{2x}$

4.6 $(D^4 + 4)y = 0$ **ANS** $y_C = e^x(A_1 \cos x + B_1 \sin x) + e^{-x}(A_2 \cos x + B_2 \sin x)$

4.7 $(D^4 + 18D^2 + 81)y = 0$ **ANS** $y_C = (A_0 + A_1x) \cos 3x + (B_0 + B_1x) \sin 3x$

4.8 $(D^4 - 4D^2 + 16)y = 0$

ANS $y_C = e^{\sqrt{3}x}(A_1 \cos x + B_1 \sin x) + e^{-\sqrt{3}x}(A_2 \cos x + B_2 \sin x)$

4.9 $(D^4 - 2D^3 + 2D^2 - 2D + 1)y = 0$ **ANS** $y_C = (C_0 + C_1x)e^x + A \cos x + B \sin x$

4.10 $(D^4 - 5D^3 + 5D^2 + 5D - 6)y = 0$

ANS $y_C = C_1e^{-x} + C_2e^x + C_3e^{2x} + C_4e^{3x}$

4.11 $(D^5 - 6D^4 + 9D^3)y = 0$ **ANS** $y_C = C_0 + C_1x + C_2x^2 + (D_0 + D_1x)e^{3x}$

4.12 $(D^6 - 64)y = 0$ **ANS** $y_C = C_1e^{-2x} + C_2e^{2x}$

$+ e^{-x}[A_1 \cos(\sqrt{3}x) + B_1 \sin(\sqrt{3}x)] + e^x[A_2 \cos(\sqrt{3}x) + B_2 \sin(\sqrt{3}x)]$

Belirsiz katsayılar yöntemini kullanarak aşağıdaki homojen olmayan (ikinci yanlı) adi diferansiyel denklemlerin özel çözümlerinin formunu belirleyiniz.

$$4.13 \quad (D^2 + 6D + 10)y = 3xe^{-3x} - 2e^{3x} \cos x$$

$$4.14 \quad (D^2 - 8D + 17)y = e^{4x}(x^2 - 3x \sin x)$$

$$4.15 \quad (D^2 - 2D + 2)y = (x + e^x) \sin x$$

$$4.16 \quad (D^2 + 4)y = \sinh x \sin 2x$$

$$4.17 \quad (D^2 + 2D + 2)y = \cosh x \sin x$$

$$4.18 \quad (D^3 + D)y = \sin x + x \cos x$$

$$4.19 \quad (D^3 - 2D^2 + 4D - 8)y = e^{2x} \sin 2x + 2x^2$$

$$4.20 \quad (D^3 - 4D^2 + 3D)y = x^2 + xe^{2x}$$

$$4.21 \quad (D^4 + D^2)y = 7x - 3 \cos x$$

$$4.22 \quad (D^4 + 5D^2 + 4)y = \sin x \cos 2x$$

Belirsiz katsayılar yöntemini kullanarak aşağıdaki homojen olmayan (ikinci yanlı) adi diferansiyel denklemlerin özel çözümlerini bulunuz.

$$4.23 \quad (D^5 - 3D^3 + 1)y = 9e^{2x} \quad \text{ANS} \quad y_p = e^{2x}$$

$$4.24 \quad (D - 1)^3 y = 48xe^x \quad \text{ANS} \quad y_p = 2x^4 e^x$$

$$4.25 \quad (D^3 - 3D)y = 9x^2 \quad \text{ANS} \quad y_p = -x^3 - 2x$$

$$4.26 \quad (D^5 + 4D^3)y = 7 + x \quad \text{ANS} \quad y_p = \frac{1}{96}x^3(28 + x)$$

$$4.27 \quad (D^2 - D - 2)y = 36xe^{2x} \quad \text{ANS} \quad y_p = 2e^{2x}(3x^2 - 2x)$$

$$4.28 \quad (D^4 + 16)y = 64 \cos 2x \quad \text{ANS} \quad y_p = 2 \cos 2x$$

$$4.29 \quad (D^4 + 4D^2 - 1)y = 44 \sin 3x \quad \text{ANS} \quad y_p = \sin 3x$$

$$4.30 \quad (D^3 + D^2 + 5D + 5)y = 5 \cos 2x \quad \text{ANS} \quad y_p = 2 \sin 2x + \cos 2x$$

$$4.31 \quad (D^2 + 3D + 5)y = 5e^{-x} \sin 2x \quad \text{ANS} \quad y_p = -e^{-x}(2 \cos 2x + \sin 2x)$$

$$4.32 \quad (D^4 - 1)y = 4e^{-x} \quad \text{ANS} \quad y_p = -xe^{-x}$$

$$4.33 \quad (D^2 + 4)y = 8 \sin^2 x \quad \text{ANS} \quad y_p = 1 - x \sin 2x$$

$$4.34 \quad (D^3 - D^2 + D - 1)y = 4 \sin x \quad \text{ANS} \quad y_p = x(\cos x - \sin x)$$

$$4.35 \quad (D^4 - D^2)y = 2e^x \quad \text{ANS} \quad y_p = xe^x$$

Belirsiz katsayılar yöntemini kullanarak aşağıdaki homojen olmayan (ikinci yanlı) adi diferansiyel denklemlerin genel çözümlerini bulunuz.

$$4.36 \quad y'' - 4y' + 4y = (1+x)e^x + 2e^{2x} + 3e^{3x}$$

$$\text{ANS} \quad y = (C_0 + C_1x)e^{2x} + (x+3)e^x + x^2e^{2x} + 3e^{3x}$$

$$4.37 \quad (D^2 - 2D + 5)y = 4e^x \cos 2x$$

$$\text{ANS} \quad y = e^x(A \cos 2x + B \sin 2x) + xe^x \sin 2x$$

$$4.38 \quad (D^2 + 4)y = 4 \sin 2x \quad \text{ANS} \quad y = A \cos 2x + B \sin 2x - x \cos 2x$$

$$4.39 \quad (D^2 - 1)y = 12x^2e^x + 3e^{2x} + 10 \cos 3x$$

$$\text{ANS} \quad y_p = C_1e^{-x} + C_2e^x + e^x(2x^3 - 3x^2 + 3x) + e^{2x} - \cos 3x$$

$$4.40 \quad y'' + y = 2 \sin x - 3 \cos 2x \quad \text{ANS} \quad y = A \cos x + B \sin x - x \cos x + \cos 2x$$

$$4.41 \quad y'' - y' = e^x(10 + x^2) \quad \text{ANS} \quad y = C_1 + C_2e^x + e^x\left(\frac{1}{3}x^3 - x^2 + 12x\right)$$

$$4.42 \quad (D^2 - 4)y = 96x^2e^{2x} + 4e^{-2x}$$

$$\text{ANS} \quad y = C_1e^{-2x} + C_2e^{2x} + e^{2x}(8x^3 - 6x^2 + 3x) - xe^{-2x}$$

$$4.43 \quad (D^2 + 2D + 2)y = 5 \cos x + 10 \sin 2x$$

$$\text{ANS} \quad y = e^{-x}(A \cos x + B \sin x) + \cos x + 2 \sin x - 2 \cos 2x - \sin 2x$$

$$4.44 \quad (D^2 - 2D + 2)y = 4x - 2 + 2e^x \sin x$$

$$\text{ANS} \quad y = e^x(A \cos x + B \sin x) + 2x + 1 - xe^x \cos x$$

$$4.45 \quad (D^2 - 4D + 4)y = 4xe^{2x} \sin 2x$$

$$\text{ANS} \quad y = (C_0 + C_1x)e^{2x} - e^{2x}(x \sin 2x + \cos 2x)$$

$$4.46 \quad (D^3 - D^2 + D - 1)y = 15 \sin 2x$$

$$\text{ANS} \quad y = Ce^x + A \cos x + B \sin x + 2 \cos 2x + \sin 2x$$

$$4.47 \quad (D^3 + 3D^2 - 4)y = 40 \sin 2x$$

$$\text{ANS} \quad y = (C_0 + C_1x)e^{-2x} + C_2e^x + \cos 2x - 2 \sin 2x$$

$$4.48 \quad y''' - y'' + y' - y = 2e^x + 5e^{2x}$$

$$\text{ANS} \quad y = A \cos x + B \sin x + Ce^x + xe^x + e^{2x}$$

$$4.49 \quad (D^3 - 6D^2 + 11D - 6)y = 10e^x \sin x$$

$$\text{ANS} \quad y = C_1e^x + C_2e^{2x} + C_3e^{3x} + e^x(3 \sin x - \cos x)$$

$$4.50 \quad (D^3 - 2D - 4)y = 50(\sin x + e^{2x})$$

$$\text{ANS} \quad y = Ce^{2x} + e^{-x}(A \cos x + B \sin x) + 6 \cos x - 8 \sin x + 5xe^{2x}$$

$$4.51 \quad y''' - 3y'' + 4y = 12e^{2x} + 4e^{3x}$$

$$\text{ANS} \quad y = (C_0 + C_1x)e^{2x} + C_3e^{-x} + 2x^2e^{2x} + e^{3x}$$

$$4.52 \quad (D^4 - 8D^2 + 16)y = 32e^{2x} + 16x^3$$

$$\text{ANS} \quad y = (C_0 + C_1x)e^{-2x} + (D_0 + D_1x)e^{2x} + x^2e^{2x} + x^3 + 3x$$

$$4.53 \quad (D^4 - 18D^2 + 81)y = 72e^{3x} + 729x^2$$

$$\text{ANS} \quad y = (C_0 + C_1x)e^{-3x} + (D_0 + D_1x)e^{3x} + x^2e^{3x} + 9x^2 + 4$$

Sabitlerin Değişimi Yöntemi

$$4.54 \quad y'' - y = x^{-1} - 2x^{-3} \quad \text{ANS} \quad y = C_1 e^{-x} + C_2 e^x - x^{-1}$$

$$4.55 \quad y'' - y = \frac{1}{\sinh x} \quad \text{ANS} \quad y = C_1 e^{-x} + C_2 e^x - x e^{-x} + \sinh x \ln|1 - e^{-2x}|$$

$$4.56 \quad y'' - 2y' + y = \frac{e^x}{x} \quad \text{ANS} \quad y = (C_0 + C_1 x + x \ln|x|) e^x$$

$$4.57 \quad y'' + 3y' + 2y = \sin e^x \quad \text{ANS} \quad y = C_1 e^{-2x} + C_2 e^{-x} - e^{-2x} \sin e^x$$

$$4.58 \quad y'' - 3y' + 2y = \sin e^{-x} \quad \text{ANS} \quad y = C_1 e^x + C_2 e^{2x} - e^{2x} \sin e^{-x}$$

$$4.59 \quad y'' + y = \sec^3 x \quad \text{ANS} \quad y = A \cos x + B \sin x + \frac{1}{2} \sec x$$

$$4.60 \quad y'' - y = (1 - e^{2x})^{-\frac{1}{2}} \quad \text{ANS} \quad y = C_1 e^{-x} + C_2 e^x - \frac{1}{2} e^{-x} \sin^{-1} e^x - \frac{1}{2} \sqrt{1 - e^{2x}}$$

$$4.61 \quad y'' - y = e^{-2x} \sin e^{-x} \quad \text{ANS} \quad y = C_1 e^{-x} + C_2 e^x - \sin e^{-x} - e^x \cos e^{-x}$$

$$4.62 \quad y'' + 2y' + y = 15e^{-x} \sqrt{x+1} \quad \text{ANS} \quad y = e^{-x} [C_0 + C_1 x + 4(x+1)^{\frac{5}{2}}]$$

$$4.63 \quad y'' + 4y = 2 \tan x$$

$$\text{ANS} \quad y = A \cos 2x + B \sin 2x + \sin 2x \ln|\cos x| - x \cos 2x$$

$$4.64 \quad y'' - 2y' + y = \frac{e^{2x}}{(e^x + 1)^2} \quad \text{ANS} \quad y = (C_0 + C_1 x) e^x + e^x \ln(1 + e^x)$$

$$4.65 \quad y'' + y' = \frac{1}{1 + e^x} \quad \text{ANS} \quad y = C_1 + C_2 e^{-x} - \ln(e^{-x} + 1) - e^{-x} \ln(e^x + 1)$$

Euler (Cauchy-Euler) Diferansiyel Denklemleri

4.66 $(x^2 D^2 - xD + 1)y = \ln x$ **ANS** $y = (C_0 + C_1 \ln x)x + 2 + \ln x$

4.67 $x^2 y'' + 3xy' + 5y = \frac{5}{x^2} \ln x$

ANS $y = x^{-1} [A \cos(2 \ln x) + B \sin(2 \ln x)] + x^{-2} \left(\frac{2}{5} + \ln x \right)$

4.68 $(x^3 D^3 + 2x^2 D^2 - xD + 1)y = 9x^2 \ln x$

ANS $y_C = C_1 x^{-1} + (C_2 + C_3 \ln x)x + (3 \ln x - 7)x^2$

4.69 $[(x-2)^2 D^2 - 3(x-2)D + 4]y = x$

ANS $y_C = (x-2)^2 (C_0 + C_1 \ln|x-2|) + x - \frac{3}{2}$

4.70 $x^3 y''' + 3x^2 y'' + xy' - y = x^2$

ANS $y = Cx + \frac{1}{\sqrt{x}} \left[A \cos\left(\frac{\sqrt{3}}{2} \ln x\right) + B \sin\left(\frac{\sqrt{3}}{2} \ln x\right) \right] + \frac{x^2}{7}$