Laplace Dönüşümü ve Uygulamaları

Alıştırmalar

1. Aşağıdaki fonksiyonların Laplace dönüşümlerini bulunuz.

6.1 $f(t) = 4t^3 - 2t^2 + 5$ ANS $F(s) = \dfrac{24 - 4s + 5s^3}{s^4}$

6.2 $f(t) = 3\sin 2t - 4\cos 5t$ ANS $F(s) = \dfrac{6}{s^2+4} - \dfrac{4s}{s^2+25}$

6.3 $f(t) = e^{-2t}(4\cos 3t + 5\sin 3t)$ ANS $F(s) = \dfrac{4s+23}{s^2+4s+13}$

6.4 $f(t) = 3\cosh 6t + 8\sinh 3t$ ANS $F(s) = \dfrac{3s}{s^2-36} + \dfrac{24}{s^2-9}$

6.5 $f(t) = 3t\cos 2t + t^2 e^t$ ANS $F(s) = \dfrac{3(s^2-4)}{(s^2+4)^2} + \dfrac{2}{(s-1)^3}$

6.6 $f(t) = t\cosh 2t + t^2 \sin 5t + t^3$ ANS $F(s) = \dfrac{s^2+4}{(s^2-4)^2} + \dfrac{10(3s^2-25)}{(s^2+25)^3} + \dfrac{6}{s^4}$

6.7 $f(t) = 7e^{-5t}\cos 2t + 9\sinh^2 2t$ ANS $F(s) = \dfrac{7(s+5)}{(s+5)^2+4} + \dfrac{72}{s(s^2-16)}$

6.8 $f(t) = \begin{cases} 0, & t < \pi, \\ \sin t, & t > \pi. \end{cases}$ ANS $F(s) = -\dfrac{e^{-\pi s}}{s^2+1}$

6.9 $f(t) = \begin{cases} 0, & t < 1, \\ 4t^2+3t-8, & t > 1. \end{cases}$ ANS $F(s) = e^{-s}\left(\dfrac{8}{s^3} + \dfrac{11}{s^2} - \dfrac{1}{s}\right)$

6.10 $f(t) = \begin{cases} 0, & t < 1, \\ t^2 - 1, & 1 < t < 2, \\ 0, & t > 2. \end{cases}$ ANS $F(s) = \dfrac{2(s+1)e^{-s} - (3s^2+4s+2)e^{-2s}}{s^3}$

6.11 $f(t) = \begin{cases} \sin t, & t < \pi, \\ 4\sin 3t, & t > \pi. \end{cases}$ ANS $F(s) = \dfrac{1}{s^2+1} - \dfrac{e^{-\pi s}(11s^2+3)}{(s^2+1)(s^2+9)}$

6.12 $f(t) = \begin{cases} 2t, & 0 < t < 2, \\ 2 + t, & 2 < t < 4, \\ 10 - t, & 4 < t < 10, \\ 0, & t > 10. \end{cases}$ ANS $F(s) = \dfrac{2 - e^{-2s} - 2e^{-4s} + e^{-10s}}{s^2}$

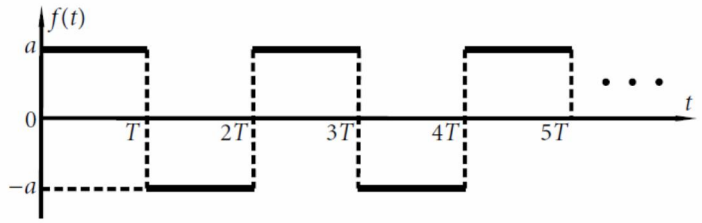
6.13 $f(t) = t^3\,\delta(t-2) + 3\cos 5t\,\delta(t-\pi)$ **ANS** $F(s) = 8\mathrm{e}^{-2s} - 3\mathrm{e}^{-\pi s}$

6.14 $f(t) = \sinh 4t\,\delta(t+2) + \mathrm{e}^{2t}\,\delta(t-1) + t^2\mathrm{e}^{-3t}\,\delta(t-2) + \cos \pi t\,\delta(t-3)$

ANS $F(s) = \mathrm{e}^{2-s} + 4\mathrm{e}^{-6-2s} - \mathrm{e}^{-3s}$

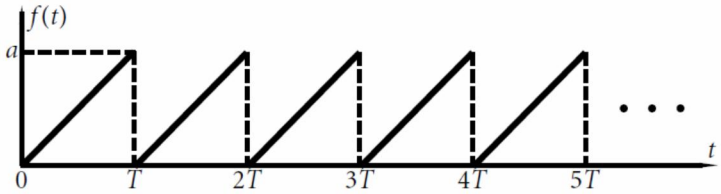
2. Aşağıdaki periyodik fonksiyonları "Birim Basamak Fonksiyonu" veya "Dirac Delta Fonksiyonu" türünden ifade ediniz ve Laplace dönüşümlerini bulunuz.

6.15



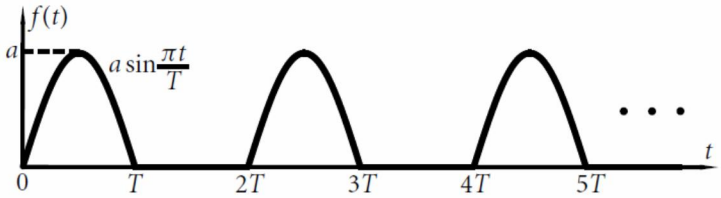
ANS $F(s) = \dfrac{a}{s}\left\{1 + 2\sum_{n=1}^{\infty}(-1)^n\mathrm{e}^{-nTs}\right\}$

6.16



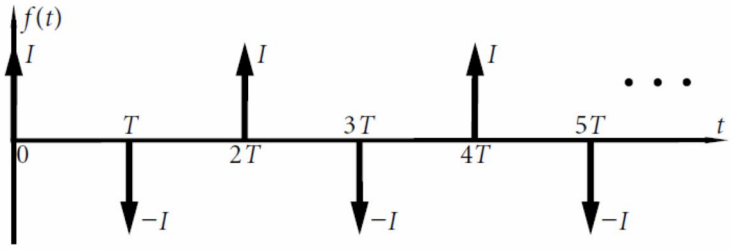
ANS $F(s) = \dfrac{a}{Ts^2} - \dfrac{a}{s}\sum_{n=0}^{\infty}\mathrm{e}^{-(n+1)Ts}$

6.17



ANS $F(s) = \dfrac{aT\pi}{T^2s^2 + \pi^2}\sum_{n=0}^{\infty}\mathrm{e}^{-nTs}$

6.18



ANS $F(s) = I\cdot\sum_{n=0}^{\infty}(-1)^n\mathrm{e}^{-nTs}$

3. Aşağıdaki fonksiyonların ters Laplace dönüşümlerini bulunuz.

6.19 $F(s) = \dfrac{s}{(s+1)^3}$ ANS $f(t) = \left(t - \frac{1}{2}t^2\right)e^{-t}$

6.20 $F(s) = \dfrac{4(2s+1)}{s^2-2s-3}$ ANS $f(t) = 7e^{3t} + e^{-t}$

6.21 $F(s) = \dfrac{3s+2}{s^2+6s+10}$ ANS $f(t) = e^{-3t}(3\cos t - 7\sin t)$

6.22 $F(s) = \dfrac{3s^2+2s-1}{s^2-5s+6}$ ANS $f(t) = 3\delta(t) + 32e^{3t} - 15e^{2t}$

6.23 $F(s) = \dfrac{30}{(s^2+1)(s^2-9)}$ ANS $f(t) = -3\sin t + \sinh 3t$

6.24 $F(s) = \dfrac{13s}{(s^2-4)(s^2+9)}$ ANS $f(t) = -\cos 3t + \cosh 2t$

6.25 $F(s) = \dfrac{40s}{(s+1)(s+2)(s^2-9)}$ ANS $f(t) = 10e^{-3t} - 16e^{-2t} + 5e^{-t} + e^{3t}$

6.26 $F(s) = \dfrac{2}{s^3(s^2+1)}$ ANS $f(t) = 2\cos t + t^2 - 2$

6.27 $F(s) = \dfrac{s}{(s+1)(s+2)^3}$ ANS $f(t) = (t^2+t+1)e^{-2t} - e^{-t}$

6.28 $F(s) = \dfrac{8}{(s-1)(s+1)^2(s^2+1)}$ ANS $f(t) = e^t - (2t+3)e^{-t} + 2\cos t - 2\sin t$

6.29 $F(s) = \dfrac{162}{s^3(s^2-9)}$ ANS $f(t) = -9t^2 - 2 + 2\cosh 3t$

4. Aşağıdaki fonksiyonların ters Laplace dönüşümlerini konvolüsyon integralini kullanarak bulunuz.

6.30 $Y(s) = \dfrac{1}{s(s^2+a^2)^2}$ ANS $y(t) = \dfrac{1}{2a^4}(2 - 2\cos at - at\sin at)$

6.31 $Y(s) = \dfrac{1}{s^2(s^2+a^2)^2}$ ANS $y(t) = \dfrac{1}{2a^5}(2at + at\cos at - 3\sin at)$

6.32 $Y(s) = \dfrac{4}{s(s^2+4s+4)}$ ANS $y(t) = 1 - (2t+1)e^{-2t}$

6.33 $Y(s) = \dfrac{16}{s^3(s^2+4s+4)}$ ANS $y(t) = -(2t+3)e^{-2t} + 2t^2 - 4t + 3$

6.34 $Y(s) = \dfrac{6}{s(s^2+4s+3)}$ ANS $y(t) = -3e^{-t} + e^{-3t} + 2$

6.35 $Y(s) = \dfrac{5}{s(s^2+4s+5)}$ ANS $y(t) = -e^{-2t}(2\sin t + \cos t) + 1$

5. Aşağıdaki diferansiyel denklemleri Laplace dönüşümünü kullanarak çözünüz.

6.36 $y'' + 4y' + 3y = 60\cos 3t, \quad y(0) = 1, \quad y'(0) = -1$

ANS $y(t) = 5e^{-3t} - 2e^{-t} - 2\cos 3t + 4\sin 3t$

6.37 $y'' + y' - 2y = 9e^{-2t}, \; y(0) = 3, \; y'(0) = -6$ ANS $y(t) = e^{t} - (3t-2)e^{-2t}$

6.38 $y'' - y' - 2y = 2t^2 + 1, \quad y(0) = 6, \quad y'(0) = 2$

ANS $y(t) = 5e^{-t} + 3e^{2t} - t^2 + t - 2$

6.39 $y'' + 4y = 8\sin 2t, \quad y(0) = 1, \quad y'(0) = 4$

ANS $y(t) = (-2t+1)\cos 2t + 3\sin 2t$

6.40 $y'' - 2y' + y = 4e^{-t} + 2e^{t}, \quad y(0) = -1, \quad y'(0) = 2$

ANS $y(t) = e^{-t} + (t^2+5t-2)e^{t}$

6.41 $y'' - 2y' + 2y = 8e^{-t}\sin t, \quad y(0) = 1, \quad y'(0) = -1$

ANS $y(t) = -e^{t}\sin t + e^{-t}(\cos t + \sin t)$

6.42 $y'' - 2y' + 5y = 8e^{t}\sin 2t, \quad y(0) = 1, \quad y'(0) = -1$

ANS $y(t) = -(2t-1)e^{t}\cos 2t$

6.43 $y'' + y' - 2y = 54te^{-2t}, \quad y(0) = 6, \quad y'(0) = 0$

ANS $y(t) = -(9t^2+6t)e^{-2t} + 6e^{t}$

6.44 $y'' - y' - 2y = 9e^{2t}H(t-1), \quad y(0) = 6, \quad y'(0) = 0$

ANS $y(t) = 4e^{-t} + 2e^{2t} + \left[(3t-4)e^{2t} + e^{3-t}\right]H(t-1)$

6.45 $y'' + 2y' + y = 2\sin t\, H(t-\pi), \quad y(0) = 1, \quad y'(0) = 0$

ANS $y(t) = (t+1)e^{-t} - \left[\cos t + (t+1-\pi)e^{\pi-t}\right]H(t-\pi)$

6.46 $y'' + 4y = 8\sin 2t\, H(t-\pi), \quad y(0) = 0, \quad y'(0) = 2$

ANS $y(t) = \sin 2t + \left[2(\pi-t)\cos 2t + \sin 2t\right]H(t-\pi)$

6.47 $y'' + 4y = 8(t^2+t-1)H(t-2), \quad y(0) = 1, \quad y'(0) = 2$

ANS $y(t) = \sin 2t + \cos 2t + \left[2t^2+2t-3-9\cos(2t-4)-5\sin(2t-4)\right]H(t-2)$

6.48 $y'' - 3y' + 2y = e^{t}H(t-2), \quad y(0) = 1, \quad y'(0) = 2$

ANS $y(t) = e^{2t} + \left[(1-t)e^{t} + e^{2t-2}\right]H(t-2)$

6.49 $y'' - 5y' + 6y = \delta(t-2), \quad y(0) = -1, \quad y'(0) = 1$

ANS $y(t) = -4e^{2t} + 3e^{3t} + \left[e^{3(t-2)} - e^{2(t-2)}\right]H(t-2)$

6.50 $y'' + 4y = 4H(t-\pi) + 2\delta(t-\pi), \quad y(0) = -1, \ y'(0) = 2$

ANS $y(t) = \sin 2t - \cos 2t + (1 + \sin 2t - \cos 2t)\,H(t-\pi)$

6.51 $y''' - y'' + 4y' - 4y = 10\mathrm{e}^{-t}, \quad y(0) = 5, \ y'(0) = -2, \ y''(0) = 0$

ANS $y(t) = -\mathrm{e}^{-t} + 5\mathrm{e}^{t} + \cos 2t - 4\sin 2t$

6.52 $y'''' - 5y'' + 4y = 120\mathrm{e}^{3t}\,H(t-1), \ y(0) = 15, \ y'(0) = -6, \ y''(0) = 0,$
$y'''(0) = 0$ ANS $y(t) = 6\mathrm{e}^{t} + 14\mathrm{e}^{-t} - 2\mathrm{e}^{2t} - 3\mathrm{e}^{-2t} + (10\mathrm{e}^{t+2} - 5\mathrm{e}^{-t+4}$
$-10\mathrm{e}^{2t+1} + 2\mathrm{e}^{-2t+5} + 3\mathrm{e}^{3t})\,H(t-1)$

6.53 $y'''' + 3y'' - 4y = 40t^2\,H(t-2), \quad y(0) = y'(0) = y''(0) = y'''(0) = 0$

ANS $y(t) = \left[-10t^2 - 15 + 40\mathrm{e}^{t-2} + 8\mathrm{e}^{2-t} + 7\cos(2t-4) + 4\sin(2t-4)\right]H(t-2)$

6.54 $y'''' + 4y = (2t^2 + t + 1)\,\delta(t-1), \quad y(0) = 1, \ y'(0) = -2, \ y''(0) = 0,$
$y'''(0) = 0$ ANS $y(t) = \mathrm{e}^{-t}\cos t - \sin t \cosh t + \big[\sin(t-1)\cosh(t-1)$
$-\cos(t-1)\sinh(t-1)\big]H(t-1)$