1 a) $\mathcal{L}\{e^{-3t}\} = \frac{1}{s+3}$, $\mathcal{L}\left\{\int_0^t e^{-3u}du\right\} = \frac{1}{s(s+3)}$ bulunur

1 b) $\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2-2s+1+4}\right\} = \mathcal{L}^{-1}\left\{\frac{e^{-3s}}{(s-1)^2+4}\right\} = \frac{1}{2}e^{t-3}\sin 2(t-3)u(t-3)$

2) $s^2Y(s) - sy(0) - y'(0) + m^2Y(s) = \frac{s}{s^2+4} \Rightarrow (s^2+m^2)Y(s) = \frac{s}{s^2+4} + s + 2 \Rightarrow Y(s) = \frac{s}{(s^2+m^2)(s^2+4)} + \frac{s}{s^2+m^2} + \frac{2}{s^2+m^2}$

$\Rightarrow Y(s) = \frac{1}{m^2-4}\left(\frac{s}{s^2+4} - \frac{s}{s^2+m^2}\right) + \frac{s}{s^2+m^2} + \frac{2}{s^2+m^2} \Rightarrow y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{m^2-4}\left[\cos 2t - \cos mt\right] + \cos mt + \frac{2}{m}\sin mt$ bulunur

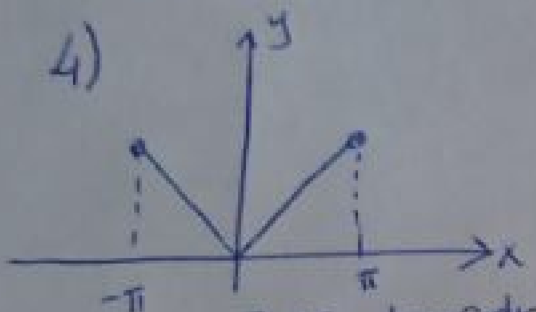
3) $-\frac{d}{ds}\left[s^2Y(s) - sy(0) - y'(0)\right] + \frac{d}{ds}\left[sY(s) - y(0)\right] + sY(s) - y(0) - Y(s) = 0$ ✓5

$-s^2Y'(s) - 2sY(s) + 5 + Y(s) + sY'(s) + sY(s) - 5 - Y(s) = 0 \Rightarrow (s^2-s)Y'(s) + sY(s) = 0$ ✓5

$\Rightarrow \frac{Y'(s)}{Y(s)} + \frac{1}{s-1} = 0 \Rightarrow \ln Y(s) + \ln(s-1) = \ln c \Rightarrow Y(s) = \frac{c}{s-1}$ ✓5 $\Rightarrow y(t) = \mathcal{L}^{-1}\{Y(s)\} = ce^t$ ✓5

$y(0) = c = 5 \Rightarrow y(t) = 5e^t$ bulunur. ✓5

4)



$P = 2\pi$ $b_n = 0$dır ✓5

$a_0 = \frac{2}{\pi}\int_0^\pi x\,dx = \frac{x^2}{\pi}\Big|_0^\pi = \pi$ ✓5

$a_n = \frac{2}{\pi}\int_0^\pi \underbrace{x}_{u}\underbrace{\cos nx\,dx}_{dv}$ $u = x \Rightarrow du = dx$, $\cos nx\,dx = dv \Rightarrow v = \frac{1}{n}\sin nx$

$a_n = \frac{2}{\pi}\left[\cancel{\frac{x\sin nx}{n}\Big|_0^\pi} - \int_0^\pi \frac{1}{n}\sin nx\,dx\right] = \frac{2}{n^2\pi}\cos nx\Big|_0^\pi = \frac{2}{n^2\pi}\left((-1)^n - 1\right)$

$n = 2m$ ise $a_{2m} = 0$, $n = 2m-1$ ise $a_{2m-1} = -\frac{4}{\pi(2m-1)^2}$ ✓5

$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{(2m-1)^2}\cos nx$ bulunur. ✓5

$x = 0$ süreklilik noktasıdır

$f(0) = 0 = \frac{\pi}{2} - \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{(2m-1)^2} \Rightarrow \frac{\pi^2}{8} = \sum_{m=1}^{\infty}\frac{1}{(2m-1)^2}$ bulunur. ✓5