Heuristics Methods for Solving the Continuous Network Design Problem

O. Baskan[1], S. Haldenbilen[1]

[1]Department of Civil Engineering, Engineering Faculty, Pamukkale University, Denizli, Turkey

Abstract

The Ant Colony Optimization (ACO) and Harmony Search (HS) algorithms to solve the Continuous Network Design Problem (CNDP) are used to tackle the optimization of signal timings with equilibrium link flows. Signal timing is defined by the cycle time and the green time for each junction and stage, respectively. Probit stochastic user equilibrium assignment is solved by way of the method of successive averages. By integrating the heuristic algorithms, traffic assignment and traffic control, the proposed Ant Colony Optimization Signal Control Algorithm (ACOSCA) and HArmony Search Signal Control Algorithm (HASCA) models solve the CNDP. The bi-level programming method is used to solve the CNDP. Test network has been chosen to illustrate the effectiveness of the proposed models and the comparisons of the values of the objective functions have been made. The results from test road network, with and without congestion, have shown the effectiveness of the proposed ACOSCA and HASCA as values of the objective function. The value of objective function obtained from the ACOSCA model was about 5% less than the value from the HASCA. Therefore, ACOSCA was chosen to implement congestion pricing to the test network. A novel approach is proposed in order to implement the congestion pricing to the test network. According to the results the improvements have been observed on the total cost of the network depending on the different demand scenarios in accordance with the base demand using link-based pricing. However, when the demand is increased more than 40% there is no improvement of the total cost since the decrease on the range of marginal cost.

Keywords: *continuous network design; heuristic methods; congestion pricing*

1 Introduction

In urban networks, traffic signals are used to control vehicle movements so as to reduce congestion, improve safety, and enable specific strategies such as minimizing delays, improving environmental pollution, etc (Teklu et al., 2007). Due to the increasing in the number of cars and developing industry, finding optimal traffic signal parameters has been an important task in order to use the network capacity optimally. There is an important interaction between the signal timings and the routes chosen by individual road users in road networks controlled by fixed time signals. The mutual interaction leads to the framework of a leader-follower or Stackelberg game, where the supplier is the leader and the user is the follower (Fisk, 1984). The signal setting problem is characterized by the so called bi-level structure. Bi-level programming problems generally are difficult to solve, because the evaluation of the upper-level objective involves solving the lower level problem for every feasible set of upper level decisions (Sun et al. 2006).

A wide range of solution methods to the signal setting problem have been discussed in the literature. Suwansirikul et al. (1987) solved equilibrium network design problem that using a direct search based on the Hooke-Jeeves' method for a small test network. Yang and Yagar (1995) used derivatives of equilibrium flows and of the corresponding travel times to solve a bi-level program for the equilibrium network design problem for a signal control optimisation. For stage length and cycle time optimization without considering offsets to minimise total travel time, Lee (1998) presented a comparison of GA and simulated annealing with iterative and local search algorithms. Ceylan and Bell (2004) proposed GA approach to solve traffic signal control and traffic assignment problem is used to tackle the optimization of signal timings with Stochastic User Equilibrium (SUE) link flows. Although proposed algorithms are capable of solving the signal timing problem for a road network, an efficient algorithm, which is capable of finding the global or near global optima of the upper level signal timing variables, is still needed. Thus, this study proposes Ant Colony Optimization Signal Control Algorithm (ACOSCA) and HArmony Search Signal Control Algorithm (HASCA) models in order to obtain better signal timings under SUE conditions. This paper is organized as follows. The basic notations are defined in the next section. Section 3 is about the formulation and bi-level programming approach. Numerical experiment is carried out in Section 4. The implementation of the congestion pricing is given in Section 5. Last section is about the conclusions.

2 Notations

c	cycle time for each junction
C_a	perceived travel time on link *a*
C_a^*	perceived travel time on link *a* with congestion price
c_a	measured travel time on link *a*
$\mathbf{h}=[h_p;\forall p\in \boldsymbol{P}_w,\forall w\in \boldsymbol{W}]$	vector of all path flows
L	number of links
$\mathbf{q}=[q_a;\forall a\in \boldsymbol{L}]$	vector of flow q_a on link *a*
$\mathbf{q}^*(\psi)$	vector of equilibrium link flows subject to signal parameters
s_a	degree of saturation on link *a*
$\mathbf{t}=[t_w;\forall w\in \boldsymbol{W}]$	vector of origin destination flows
$\boldsymbol{\varphi}$	vector of duration of green times
Ω	feasible set of signal setting variables
δ	link-path incidence matrix
Λ	OD-path incidence matrix
ϕ_a	unit price on link *a*
$\phi_{\max}$	maximum unit price on the network
ψ	signal setting variables

3 Formulation

The problem of optimising of signal setting variables $\psi=(c,\varphi)$ without considering offset term on a road network is defined as bi-level structure. The planners aim to minimise the total cost (*TC*) of a given road network on the upper level whilst the SUE link flows $\mathbf{q}^*(\psi)$ on the lower level are dealt with altering signal timings. The objective function is therefore to minimise *TC* with respect to equilibrium link flows $\mathbf{q}^*(\psi)$ subject to signal setting constraints $\psi=(c,\varphi)$. Mathematically the problem is defined as:

$$\underset{\psi\in\Omega}{Min}\;\; TC(\psi,q^*(\psi))=\sum_{a}^{L} q_a t_a(\psi,q^*(\psi)) \tag{1}$$

subject to

$$\psi=(c,\varphi)\in\Omega\;;\;\left\{\begin{array}{ll} c_{\min}\le c\le c_{\max} & \text{cycle time constraints for each junction} \\ \varphi_{\min}\le\varphi\le\varphi_{\max} & \text{green time constraints for each stage} \\ \sum_{i=1}^{m}(\varphi_i+I_i)=c & \forall m\in M \end{array}\right\}$$

where $\mathbf{q}^*(\psi)$ is implicitly defined by

$$\underset{q}{\text{Minimise}}\;\; Z(\psi,\mathbf{q})$$

$$\text{subject to}\;\; \mathbf{t}=\Lambda\mathbf{h},\; \mathbf{q}=\delta\mathbf{h},\; \mathbf{h}\ge 0$$

The ACOSCA and HASCA models are used to tackle the optimization of signal timings with stochastic equilibrium link flows. The definition of ACO and HS algorithms and its solution procedures may be obtained in Baskan et al. (2009) and Ceylan et al. (2008), respectively. Let the objective function (*TC*) takes a set of ψ signal timing variables, $\psi = (c_1, \varphi_1, \ldots\ldots\ldots, c_n, \varphi_n)$. On the assumption that each decision variable ψ can take values from a domain $\mathbf{\Omega} = [\psi_{\min}, \psi_{\max}]$ for all $\psi \in \Omega$. In order to provide the constraint of cycle time for each junction, the green timings can be distributed to the all signal stages in a road network as proposed by Ceylan and Bell (2004). The optimizing of signal timings plays a critical role to optimize the performance of the network. It is also a difficult problem because the evaluation of the upper level objective involves solving the lower level problem for every feasible set of upper level decisions. In this study, the ACOSCA and HASCA models are developed to overcome this drawback using bi-level structure. The basic flowchart of bi-level programming is given in Figure 1.

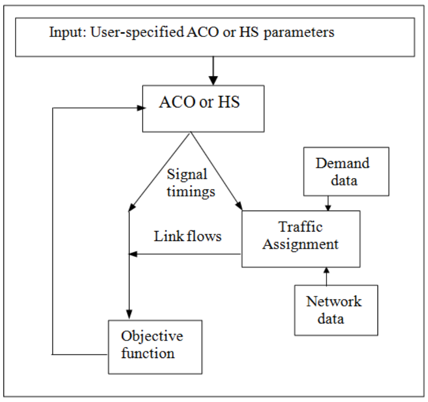

Figure 1. Flowchart of bi-level programming approach.

Probit stochastic user equilibrium (PSUE) (Sheffi, 1985) model is used to solve lower level problem so as it has the advantage of being able to represent perceptual differences in utility of alternatives on a road network. Monte-Carlo simulation method is adopted to obtain probit choice probabilities. The underlying assumption of probit model, the random error term of each alternative is assumed normally distributed. The notation $\xi \sim$ MVN (μ,Σ) indicates that the vector of error terms ξ is multivariate normal (MVN) distributed with mean vector μ and covariance matrix. The algorithm to solve PSUE may be obtained in Sheffi (1985).

4 Numerical Example

The test network is chosen that is used by Allsop and Charlesworth (1977) in order to show the performance of the ACOSCA and HASCA models to optimise signal setting variables. The network topology is given in Fig. 2, where figure is adapted from Ceylan and Bell (2004). Stage configurations and travel demands related to network can also be obtained from Ceylan (2002). This network has 20 O-D pairs and 20 signal setting variables at six signal-controlled junctions. The signal timing constraints are given as follows:

$c_{\min}$, $c_{\max}$ = 60, 100 sec   cycle time constraints for each junction

$\phi_{\min}$ = 7 sec   minimum green time for signal stages

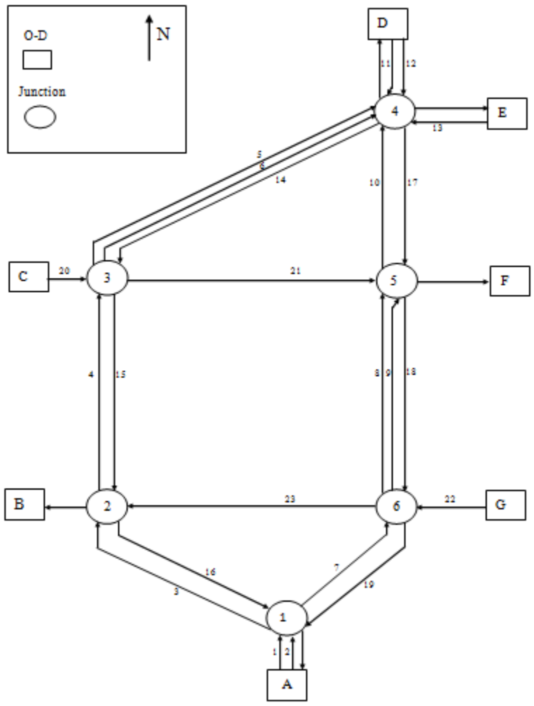

Figure 2. Layout for the test network.

The application of the ACOSCA model to the test network can be seen in Fig. 3, where the convergence of the algorithm and the evaluation of the objective function are shown. The significant improvement on the objective function takes place in the first few iteration because the ACOSCA starts with randomly generated ants in a large colony size. Finally, the number of objective function has reached to the value of 124587 veh-sec. The ACOSCA model is performed for the 297th iteration, where the difference between the values of $\boldsymbol{\psi}_k$ and $\boldsymbol{\psi}_{k+1}$ is less than 1%, and the number of objective function is obtained for that is 124587 veh-sec. The improvement rate is 7% according to the initial solution of objective function. Table 1 shows the signal timings and the final value of objective function. The final values of degree of saturation for the ACOSCA model are less than 95%.

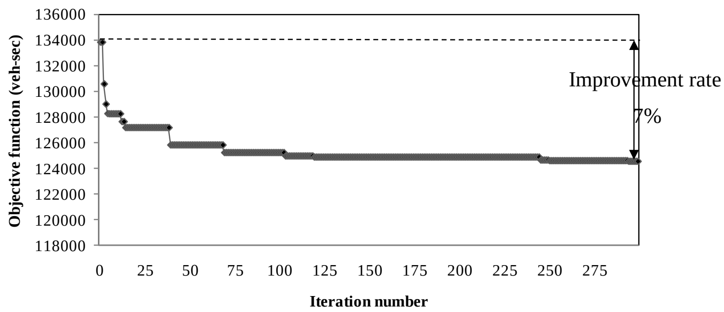

Figure 3. Convergence behaviour of the ACOSCA model.

Table 1. The final values of signal timing derived from the ACOSCA model.

Objective function (veh-sec)	Junction number	Cycle time c (sec)	Green timings in seconds		
			Stage 1	Stage 2	Stage 3
124587	1	86	7	79	-
	2	76	32	44	-
	3	69	38	31	-
	4	62	11	30	21
	5	74	11	35	28
	6	80	50	30	-

The application of the HASCA model to the same test network can be seen in Fig. 4. The number of objective function has reached to the value of 130943 veh-sec. This value is greater about 5% than the value from the ACOSCA model. The ACOSCA model was chosen to implement congestion pricing to the test network when two models were compared in terms of the value of objective function.

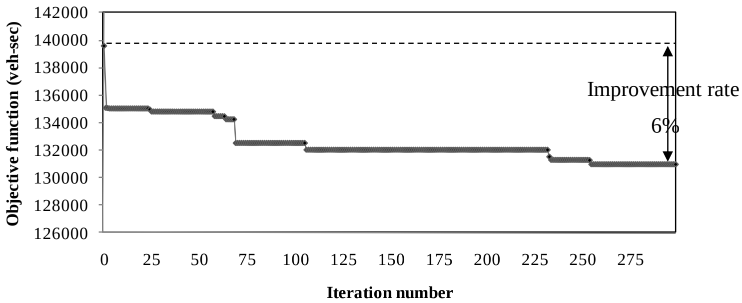

Figure 4. Convergence behaviour of the HASCA model

5 The Implementation of Congestion Pricing

Congestion pricing has been suggested for a long time as a means to solve environmental and congestion problems in urban regions. In this study, a new model was proposed to integrate congestion pricing and network design problem. The formulation is given as follows (Baskan, 2009):

$$C_a^* = C_a(1 + \frac{\phi_a}{\phi_{\max}}) \tag{2}$$

The final values of degree of saturation on all links with and without the implementation of congestion pricing are given in Table 2 according to the increased demand scenarios for 110%. The degree of saturation of some links on the network is greater that the critical value (100%) when demand increased to 110% from base demand before the implementation of congestion pricing.

As shown in Table 2, the degree of saturation of all links was decreased below its critical value when the congestion pricing was implemented. Income was obtained from optimum link prices and corresponding equilibrium link flows using following equation (Baskan, 2009).

$$G = \sum_a \phi_a * q_a \qquad \forall a \in A \tag{3}$$

where G is the income obtained after the application of congestion pricing. The obtained income according to different demand scenarios are given in Table 3.

Table 2. The final values of degree of saturation (%) without and with congestion pricing for 110% demand increase.

Before congestion pricing											
s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
35	42	36	41	62	23	50	55	94	65	90	47
s_{13}	s_{14}	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	
58	39	30	67	*100*	69	90	76	82	*112*	56	
After congestion pricing											
s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}
38	43	42	38	68	19	50	56	69	65	84	27
s_{13}	s_{14}	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	
85	54	52	69	84	49	83	84	78	94	44	

Table 3. Obtained income.

Demand increase (%)	G
110	77101
120	103482
130	106939
140	140193
150	127262

Marginal costs according to different demand scenarios are obtained using income and total cost of the network. According the results the improvements have been observed on the total cost of the network depending on the different demand scenarios according to the base demand using link-based pricing. However, when the demand is increased more than 40% there is no improvement of the total cost since the decrease on the range of marginal cost as shown in Fig. (5).

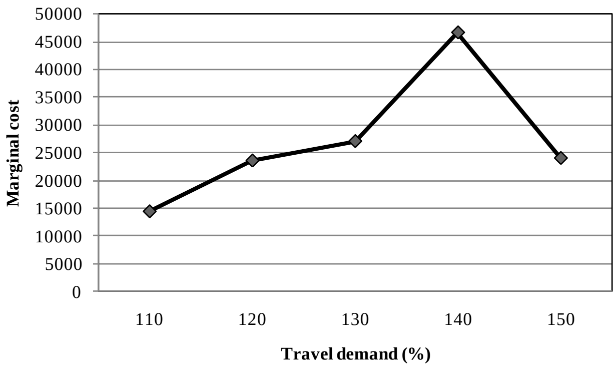

Figure 5. The variations of marginal cost according to demand scenarios

6 Conclusions

In this study, the ACOSCA and HASCA models were used to optimize signal timings on a given road network without considering offset term using bi-level programming. The Allsop and Charlesworth's test network was used to denote the performance of the models in terms of the value of objective function and degree of saturation on links. According to results, the value of objective function obtained from ACOSCA model is less than the value which was obtained from the HASCA model. The final values of degree of saturation from the ACOSCA model were less than 100% which is the critical value. The ACOSCA model was performed on PC Core2 Toshiba machine and each iteration for this test network was 12.3 sec of CPU time in Visual Basic code. On the other hand, the computation effort for the HASCA model on the same machine was carried out for each iteration in 8.5 sec of CPU time. The ACOSCA model in solving bi-level problem was found considerably successful in terms of the final value of objective function and the improvement rate according to the initial value when it was compared with the HASCA solution. Therefore, the ACOSCA was chosen to implement congestion pricing to the test network. A new formulation was proposed in order to implement the congestion pricing to the test network. According the results the improvements have been observed on the total cost of the network depending on the different demand scenarios according to the base demand using link-based pricing. However, when the demand is increased more than 40% there is no improvement of the total cost since the decrease on the range of marginal cost.

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