INDEXING INTERSECTION DELAYS WITH NUMERICAL METHODS

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ABSTRACT: This study deals with the estimation of performance indicators at signalized intersections. Generated delays are obtained with the TRANSYT traffic model. Two components of delay are modelled. For the uniform component, one form of the mathematical model is developed and two mathematical models are developed for random plus over-saturation component of total delay. Link traffic volumes and cycle times are correspondingly changed to test the model performance for oversaturation delays. Delay indexing is obtained for each change on link traffic volumes and cycle time. Proposed models are solved with quasi-Newton method to obtain weighting parameters. Results showed that with four-legs four-stage signalized intersection, the uniform and random plus over-saturation component of total delay may be estimated. Indexing delays might also possible for obtaining timing parameters and delays for given link traffic volumes. Results also showed that random plus over-saturation delay may be calculated in two steps since there is a break point between under-saturated and oversaturated links. The proposed method and delay indexing may be helpful for practitioners since all the performance and timing parameters may be obtained with timing graphs.

KEYWORDS: Delay indexing; optimization; signalized intersection

INTRODUCTION

Vehicular delay is probably the most important parameter used by transportation professionals to evaluate the performance of signalized intersections and setting up timing parameters. Delay at signalized intersections is computed as the difference between the travel time that is actually

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experienced by a vehicle while going across the intersection and the travel time this vehicle would have experienced in the absence of traffic signal control.

The vehicle delay may be divided into two groups: Uniform and random plus oversaturation delay. The uniform component consists of signal timings; random plus oversaturation component includes vehicle queuing, random arrivals and over-saturation cases of traffic flows. Kimber and Daly (1986) studied measurements of queue lengths and vehicle delays for testing the predictions of time dependent queuing models. Akcelik (1988) studied on the Highway Capacity Manual (HCM, 1985) delay formula and suggested a calibration process. Burrow (1989) recommended additional factors for the formula improved by Akcelik. Prevedouros and Koga (1996) compared HCM (1985) delay formula with that of HCM (1994). Powell (1998) proposed some correction factors representing the deceleration and acceleration delays of vehicles based on queuing to improve the HCM (1997) delay formula. Quiroga and Bullock (1999) conducted a study related to the measuring vehicle delays using Geographic Information and Global Positioning Systems. Besides, to simulate the HCM (1997) delay formula, Qiao et. Al., (2002) developed a fuzzy logic model. Dion et. al., (2004) compared various analytic models with microscopic simulation models. Akcelik and Rouphail (1993) proposed a delay model for signalized intersections that is suitable for variable demand conditions. The proposed model clarifies several issues related to the determination of the peak flow period, as well as the periods immediately preceding and following the peak. The strength of the model lies in the use of simple rules for determining flow rates within and outside the peak, using the peak flow factor, a generalization of the well-known peak hour factor parameter. A revised delay formula for the HCM is proposed. The revised formula has no constraints on the peak flow period degree of saturation, unlike the current HCM formula. Murat and Baskan (2006) are modelled vehicle delays with using artificial neural networks (ANN). ANN model compared with Webster, HCM and Akcelik delay calculation methods and field observations. Method shows encouraging results especially for the cases of over-saturation or non-uniform traffic conditions. In order to prevent the over-fitting problem, the three-way data split method was used. In this model, traffic volume, cycle time and red signal time are taken into account as significant parameters. Murat (2006) developed delay model using new approaches with Fuzzy Logic (FL) and ANN to deal with all conditions. The results of the ANN models were compared with the HCM and Akcelik's methods.

HCM (Transportation Research Board, 2000) calculates the uniform and incremental delay on signalized intersections depending on the degree of saturation and/or vehicle/capacity ratio. The incremental delay is obtained for given time period T and delay factor k. The factor k should be calibrated according to controller type on delay that is usually is difficult to determine. Thus k=0.5 is given as a default value for taking into account random arrivals and uniform service time that is equivalent to lane group capacity. This study does not need to use calibration factor k for oversaturated links since it takes IN, OUT and GO profiles of the TRANSYT traffic model for given time period T.

Dion et. al., (2004) addressed the delays at signalized intersection controlled in fixed-time and operated in a range of conditions extending from under-saturated to highly saturated. They compared the delay estimates from a deterministic queuing model, a model based on shock wave theory, the steady-state Webster model, the queue-based models defined in Australian Capacity Guide, Canadian Capacity Guide for Signalized Intersections, and the various versions of the HCM. They indicated that all delay models produced similar results for signalized intersections with low traffic demand, but that increasing differences occur as the traffic demand approaches saturation. There is no consideration were given to model components of performance indicators and no consideration for delay indexing with estimated delays. Thus, it could be better to find a different way of obtaining timing parameters at signalized intersections which should provide optimum or near optimum system performance. It is well-known that the delays at intersections are dependent on various parameters such as cycle time, stage ordering, clearance time, gap acceptance, etc.

Modelling the intersection delays with a combination of timing parameters and setting up a delay index may be useful for theoretical and practical purposes. This study therefore develops a delay models and their indexing graphs for individual signalized intersection from light to heavy traffic conditions. Proposed delay models include uniform and oversaturation components in the analysis period. Quasi-Newton method as a numerical solution is used to solve weighting parameters of proposed models.

MATHEMATICAL FORMULATIONS

Formation of the queues at intersections is given in Figure 1. The steady-state queue formation takes place when the saturation level is about less than one and the calculation of settings in this period is non-definitive. Deterministic queue growth happens when the critical level of saturation degree is greater than one. Transformation is required between under-saturation and oversaturation to obtain delays. One of well-known method for transformation may be the TRANSYT (Robertson, 1969; Vincent et al., 1980) traffic model. The formulas are given in Equation (1) and (2).

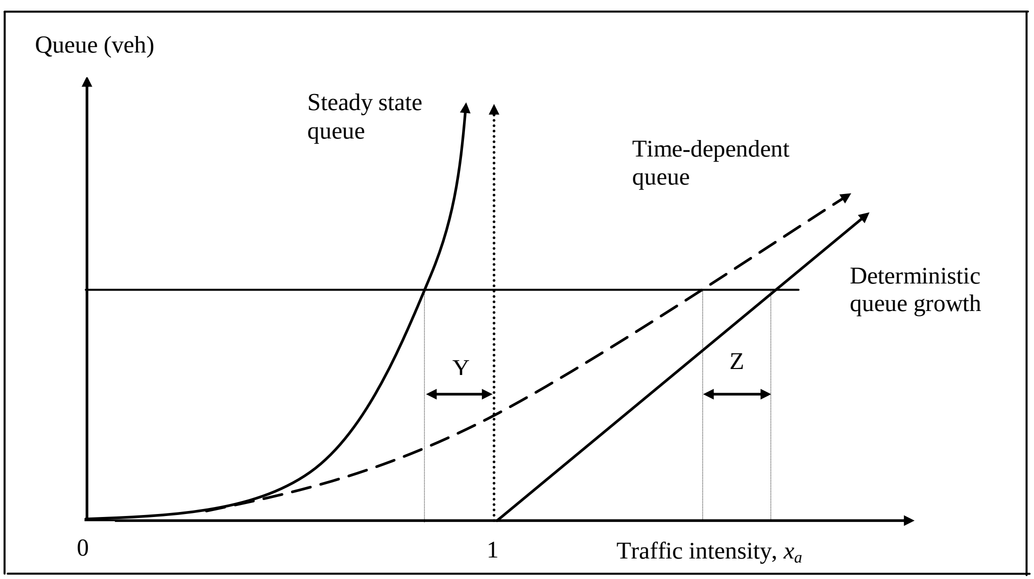

Figure 1 Time-dependent delay formulation

$$D_{aT}^{ro} = \frac{T}{4}[((q_a - \mu_a)^2 + \frac{4q_a}{T})^{0.5} + (q_a - \mu_a)] \quad (1)$$

$$d_{aT}^{ro} = \frac{D_{aT}^{ro}}{q_a} \tag{2}$$

where D_{aT}^{ro} and d_{aT}^{ro} are the TRANSYT delay formula over time period *T* and average delay to a vehicle on link *a* in ***L*** respectively, q_a is the link traffic volumes (veh/h) and μ_a uniform departure rate, and ***L*** set of links.

Delay components at signalized intersection may be analysed in following way.

Uniform component of delay: The calculation for this component for each link is carried out on the basis of whole cycles. The uniform component of delay with respect to each link *a* in ***L*** is calculated on the basis of whole time period *T*. It can be defined according to the degree of saturation for each link *a* in ***L*** as follows:

Oversaturated links with $x_a \geq 1$;
Under-saturated links with $x_a < 1$

where x_a is the degree of saturation on link *a* in ***L***.

Uniform queues and uniform delays are calculated on the basis of the difference between the cyclic cumulative departure graph and uniform departure rate μ_a for each link *a* in ***L*** in the time period *T* according to following expressions and Figure 2.

$$L_a^U = \frac{c\mu_a(1-\Lambda_a)}{2} \tag{3}$$

$$D_a^U = \frac{c\mu_a(1-\Lambda_a)}{2} \tag{4}$$

$$d_a^U = \frac{c(1-\Lambda_a)}{2} \tag{5}$$

where L_a^U is uniform queue, D_a^U is uniform delay, d_a^U is delay to a vehicle, Λ_a is proportion of green to cycle time and *c* is cycle time.

Cyclic variations of idealized arrivals and departures and corresponding delay occurrence for the delay components at a signalized intersection are given in Figure 2, where *A(t)* and *D(t)* indicate the cumulative arrivals and departures at time slice *t* on a typical signalized intersection.

For under-saturated links: It is assumed that the traffic queues develop at the start of effective red and clear at the next effective green period. It is further assumed that the cycle time, *c*, is divided into an effective green and red period. Given a junction with under-saturated conditions without the accumulation of queues, the following analytical expressions hold:

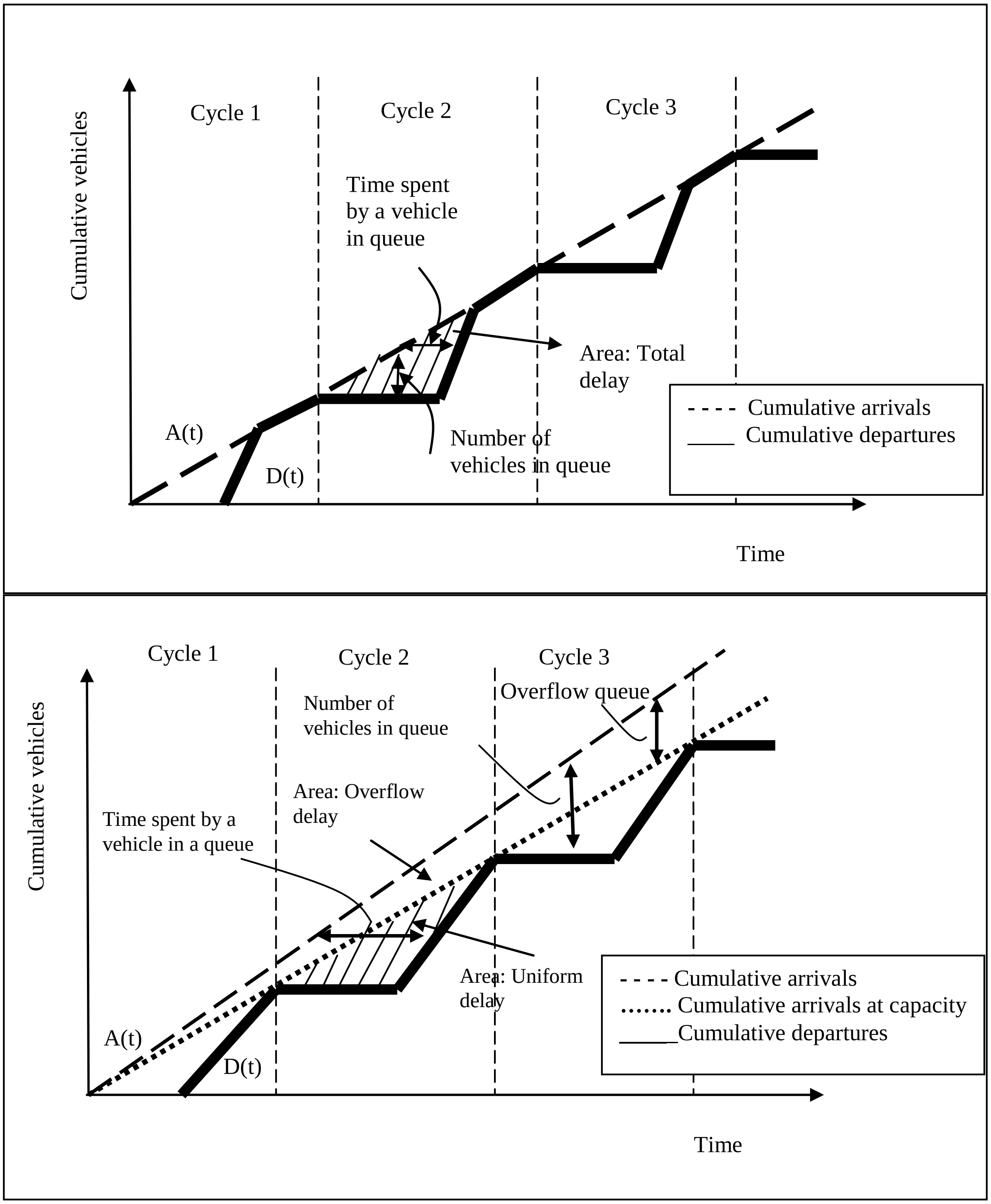

Figure 2. Typical idealized vehicle arrivals and departures at signalized intersections

$$L_a^U = \frac{x_a \mu_a (1-\Lambda_a)^2 c}{2(1-\Lambda_a x_a)} \qquad (6)$$

$$D_a^U = L_a^U \qquad (7)$$

$$d_a^U = \frac{D_a^U}{q_a} \qquad (8)$$

For random plus oversaturation (R+O) delay component of total delay, there is a need to obtain time-dependent delay calculation, which is based on the steady state and deterministic approaches. As the degree of saturation x_a approaches 1, the steady-state queue length tends to infinity. Due to the complexity of the mathematical expressions used in queuing analysis, Kimber and Hollis (1979) proposed the *coordinate transformation method*. TRANSYT (Robertson, 1969) uses the time dependent delay formula in a simulation period. This study uses the TRANSYT delay formula as in Equation (1) for the R+O of the delay since it is easy to apply.

INDEXING DELAY CALCULATIONS

Development of modelling delays at signalized intersections is applied on a simple four leg intersection with mini-circle on middle. Each approaching link consists of two lanes, through, left and right turning movements. Saturation flows are taken as 1500 veh/h for each traffic stream and assumed that it is equally distributed to each lane. The mini-circle on the middle of intersection decreases the discharching capacity of junction by about 20% from 1800 veh/h to 1500 veh/h (Ceylan et. al., 2007). Modelling and obtaining delay index are carried out with four stages. Delays are obtained for each increase on link traffic volumes. The cycle time is then optimized and the corresponding delays are noted for every increase on the traffic volumes.

An example intersection with four legs and 8 lanes can be seen in Figure 3. It has mini circle on middle, where only the stage pulses are given which does not affect the number of stages, but it affects the intersection lost time and discharching capacity. As can be seen in Figure, the circle in the middle changes the direction of move and jams the traffic which leads to decrease the lane capacity of the junctions. Note that this kind of signalized intersection is a typical in Turkey that does not work as a roundabout which is only for U-turns if they have enough queuing space. Link numbering and stage orderings are given in Figure 4.

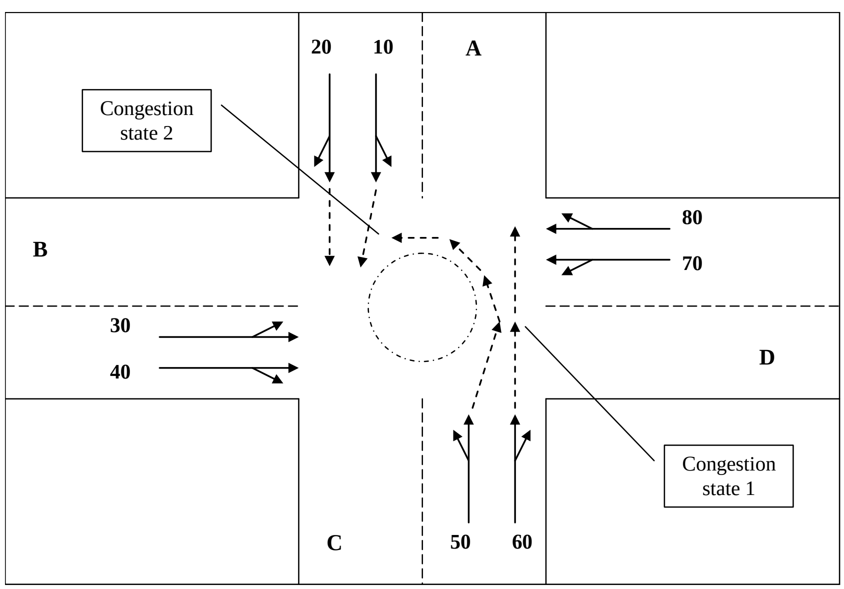

Figure 3. Typical idealized intersection with four approaching links and 8 lanes

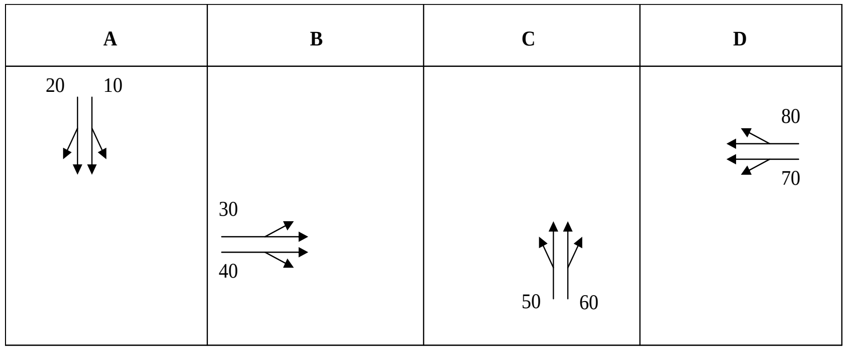

Figure 4. Example stage ordering

Flowchart of the estimation of delay components and total delay on an example network is given in Figure 5. In order to estimate delays, the program takes the values of incremental link flows and cycle time. Green timings to the stages are distributed according to the "*equisaturation*" facility by TRANSYT traffic model. After that, the TRANSYT model is called to obtain the vehicle delays until pre-specified iteration number is completed. Iteration number will be maximum of link flows or maximum of cycle time which would be given in practical applications (i.e. 180 seconds of cycle time). Therefore, maximum iteration number is set as 180. A FORTRAN 95 code is written for obtaining the generated theoretical delays at the example signalized intersection by internally calling TRANSYT traffic model using the expressions (2-8). Signal timings are noted for each run of the TRANSYT model. HCM (2000) uses different forms of the equations for obtaining delay values for oversaturated links based on incremental delay factor *k*, but this study uses coordinate transformation method and TRANSYT time dependent delay formulas. In Figure 5, the variable link flows and cycle time is considered as a counter to iterate the program for obtaining delays and their components.

For the example signalized intersection, one hour simulation period is given. The performance of the signalized intersection is obtained as uniform and random plus oversaturation delay. Typical generated junction performance values and their corresponding link saturation flows are given in Table 1. Summary of the link traffic volumes and the corresponding delays can be seen in Table 1, but all generated values are used during the modelling for ever 1 sec increase on cycle time, and 10 veh/h increase on link traffic volumes.

Using the values on Table 1, delay indexing models are developed depending on cycle time and traffic volumes. The models are broken down into two parts: Uniform delay (UD) component and random plus oversaturation (R+O) components. Form of the uniform (UD) component of total delay is given in Equation (9).

$$UD_{model} = w_1 * q^2 + w_2 * q + w_3 * c^2 + w_4 * c + w_0 \qquad (9)$$

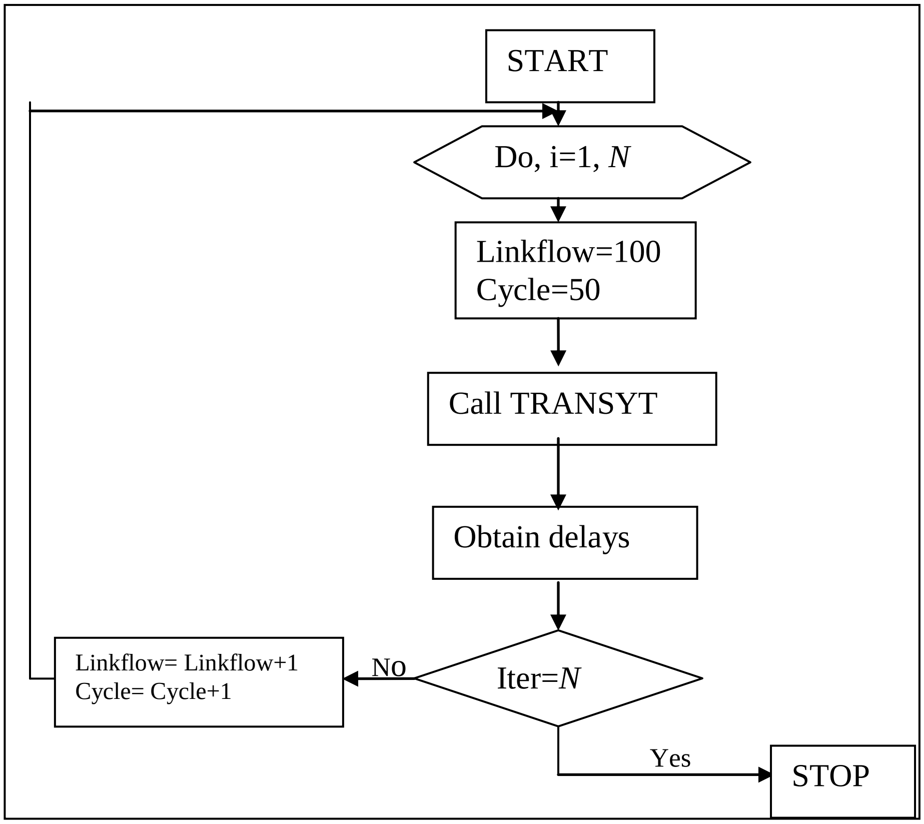

Figure 5. Obtaining generated theoretical delays via TRANSYT model

Table 1. Generated delays versus link traffic volumes and degrees of saturation

					Degree of saturation on links (%)							
***q* (veh/h)**	***c* (sec)**	**UD (veh-h/h**	**R+O (veh-h/h)**	**Total (veh-h/h)**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**
100	50	4.1	2.6	6.7	37	37	42	42	42	42	37	37
150	50	6.4	5.7	12.1	56	56	63	63	63	63	56	56
200	70	11.7	8.9	20.6	67	67	72	72	72	72	67	67
250	90	18.9	16.3	35.2	79	79	83	83	83	83	79	79
300	160	40.0	27.1	67.1	89	89	89	89	89	89	89	89
350	180	54.9	88.1	143.0	102	102	102	102	102	102	102	102
400	180	76.2	258.9	335.1	120	120	123	123	112	112	114	114
450	180	98.1	451.1	549.2	123	123	164	164	132	132	117	117
500	180	121.9	647.1	769.0	200	200	150	150	128	128	128	128
550	180	146.6	844.8	991.4	254	254	153	153	138	138	140	140
600	180	171.4	1043.6	1215.0	180	180	141	141	144	144	313	313

Fitted model for the uniform component of delay is given in Equation (10).

$$UD_{model} = 0.0005 * q^2 + 0.007 * q + 0.002 * c^2 - 0.492 * c + 16.703 \quad (10)$$

where, q is the traffic volumes (veh/h), c is cycle time (sec) and w_0, w_1,… are the model weighting parameters. Comparison of the fitted values of UD_{model} and the generated theoretical delays can be seen in Figure 6. During the solution of UD_{model}, the quasi-Newton method is used with the "solver" facility in spreadsheet. *Solver* can be used to maximize or minimize the value of a "target" worksheet cell by altering the values of other selected "changing" cells in the spreadsheet that influence the value in the target cell. It also allows constraints to be placed on the values of any cells in the worksheet. Thus, it is a general-purpose tool capable of solving constrained linear and nonlinear optimization problems.

The UD_{model} is solved with minimizing the sum of squared errors (sse) between generated theoretical delays and estimated delays for fitted equations. The form of the sse is given in the following way.

$$f(sse) = \sum_{i}^{n} (d_{generated} - d_{UDmodel})^2 \tag{11}$$

where, $d_{generated}$ is the delay values (veh-h/h) and $d_{UDmodel}$ is the model delays (veh-h/h). Minimum of the sse is obtained as 117 and $R^2 = 0.99$.

Random plus oversaturation component of total delay is modelled in two steps. In Step 1, the $R{+}O_{model}$ is developed when link traffic volumes is less than or equal to 290 veh/h. In step two, the model is developed when link traffic volumes is greater than 300 veh/h.

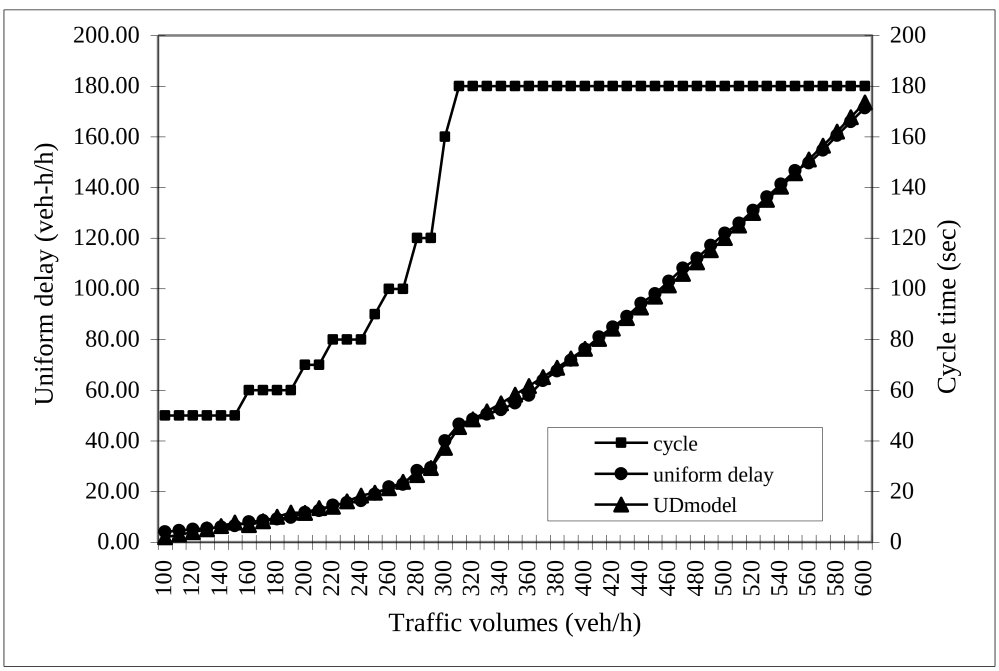

Figure 6. Comparison of UD_{model} and generated UD delays

For the step 1, following model is fitted.

$$q \leq 290 \Rightarrow RO = w_1 * q^{w_2} \tag{12}$$

For the step 2,

$$q \geq 300 \Rightarrow RO_{model} = w_1 * q^3 + w_2 * q^2 + w_3 * q + w_0 \quad (13)$$

is obtained.

Solution of the $R{+}O_{models}$ with quasi-Newton method by using solver facility, the following weighting parameters are obtained.

$$q \leq 290 \Rightarrow RG = 1.10^{-5} * q^{2.594} \qquad \text{sse=24; } R^2 = 0.98 \quad (14)$$

$$q \geq 300 \Rightarrow RG = -8.5.10^{-6} * q^3 + 0.017 * q^2 + -6.433 * q + 617.149 \qquad \text{sse=15029; } R^2 = 0.99 \quad (15)$$

Figure 7 shows the estimated R+O delays and the generated theoretical delays in an example junction. As can be seen from Figure, the delays obtaining from TRANSYT and $R{+}O_{model}$ fitted well in terms of R^2 and minimum sse.

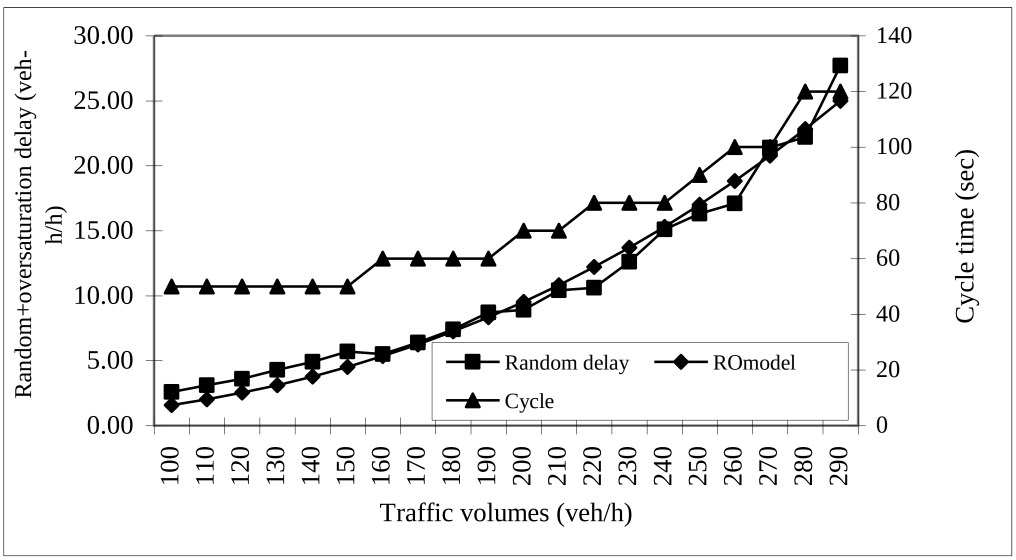

Figure 7. Comparison of modelled and generated delays ($q \leq 290$)

Figure 8 shows the estimated and fitted R+O components of total delay when $q \geq 300$.

Total delay (TD) in an example signalized junction is obtained as *TG=UG+RG* and is given in Figure 9. As can be seen in Figure 9, there is a good agreement between generated theoretical and modelled delays. The optimum cycle time will change from 50 second to 180 seconds which is obtained by TRANSYT model with CYOP facility (Vincent et. al., 1980).

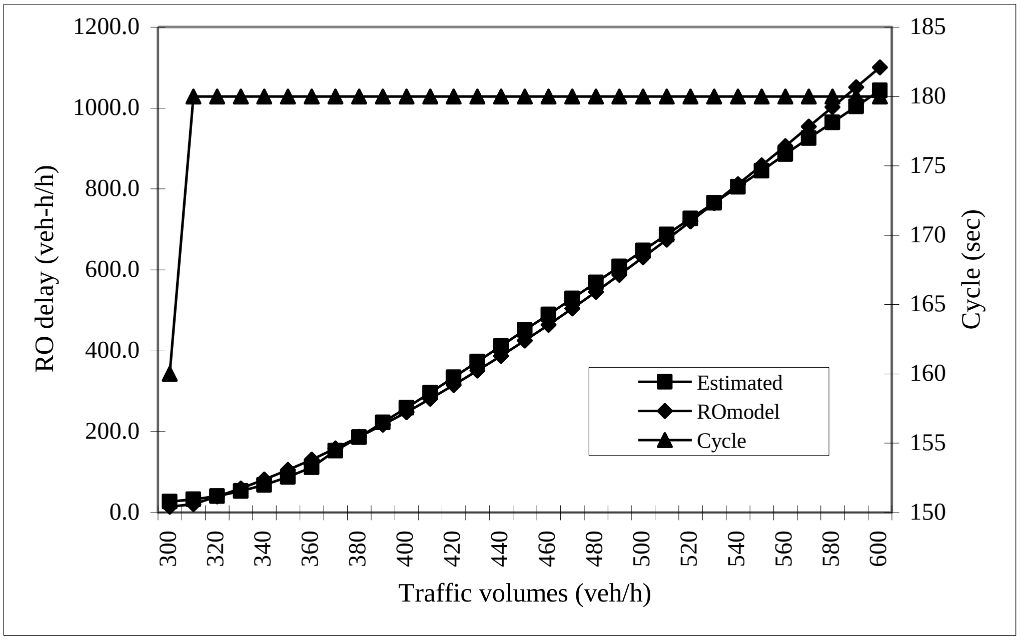

Figure 8. RO component of delay model when q ≥ 300

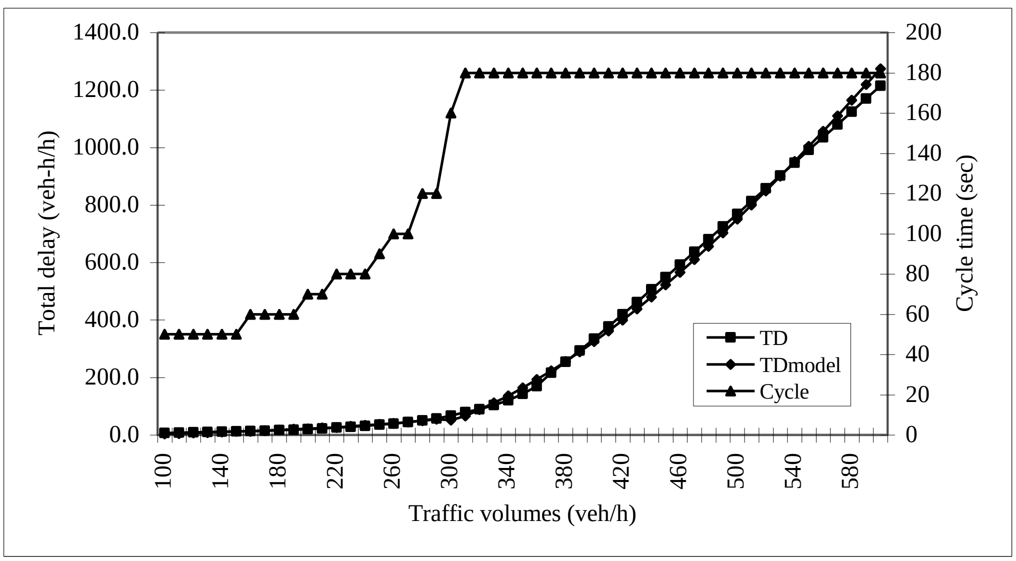

Figure 9. Comparison of total and estimated delays

Main advantage of modelling and estimating delays will provide setting up a delay index and timing parameters without calculation any complex set of equations if the simulation is given as one hour. Transformation of steady-state queue to time dependent queue may be easily seen in Figure 9. The advantage of this graph may provide practitioner to set up timing parameters and their corresponding total delays and/or components of delays. For example, if the critical links in a signalized junction will take a traffic flow of 204 veh/h, Figure 9 shows that the total delay will be about 4.20 veh-h/h and the cycle time is 60 second. After determining cycle time, distribution of green timing to stages will be easy to obtain subject to green timing constraints.

Figure 10 indicates the generated theoretical delay components with respect to link traffic volumes and cycle time during one hour simulation period. Green timings and delay components for under-saturation and oversaturation cases are obtained from TRANSYT traffic model. In Figure 10, the first y axis indicates the cycle time, the second y axis shows the delay values in a simulation period. The x axis shows the traffic volumes starting from 100 veh/h to 1400 veh/h. When link traffic volume reaches 330 per lane, the values of the degree of saturation on all links are about 97%. After that degrees of saturation in all links excess the critical value of 1. For the light traffic conditions, there are not big variations between uniform delay and random plus oversaturation delay. When the link flows are near to link capacity, the random plus over-saturation delay higher than the uniform delay. As indicated in previous studies (Chiou, 1998)), uniform delay takes place only for cyclic variations while the R+O component takes place when junction is getting congested.

Indexing or estimating performance parameters of an example junction may be read in Figure 10: If link traffic volume is 1140 veh/h that is about the link capacity, the UD is about 70 veh-h/h, R+O is 560 veh-h/h, TD is 630 veh-h/h and cycle time is 155 seconds. With this way, all the timing parameters may be obtained for measured link flows on a critical links with about equally distributed traffic volumes for all lanes in intersection approaches.

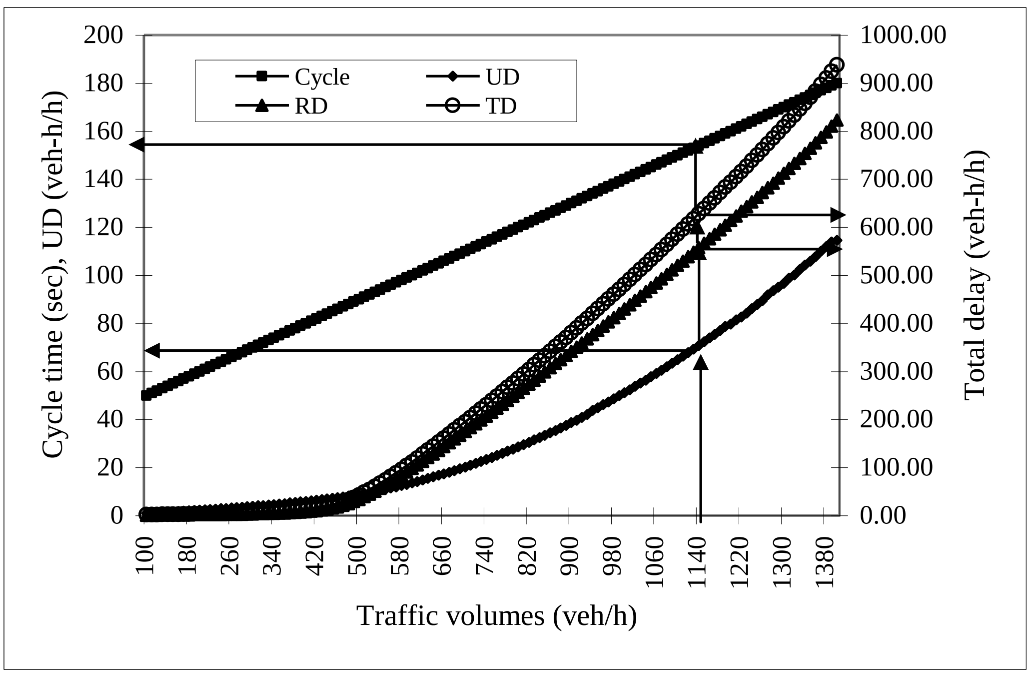

Figure 8.Indicators and timing parameter of an example junction as a whole

CONCLUSIONS

This study estimates and models the signalized intersection to set up an index for obtaining timing parameters and junction indicators. Definition of performance indicators at signalized intersections is given and corresponding performance indicators are defined and formulated. Uniform and random plus over-saturation components for total delay is analysed that allows for oversaturated links in a simulation period. TRANSYT traffic model is used to obtain the performance indicators in an example junction. TRANSYT delay formulation is used to estimate the random plus over saturation component of total delay. Link traffic volumes and cycle time are simultaneously changed. Flowchart of the solution algorithm is given. Proposed models to calculate the performance indicators of total delay are solved with quasi-Newton method with solver facility on spreadsheet. Estimation of delays is applied to a typical four leg eight lane signalized intersection. Indexing the delays and corresponding cycle times and traffic volumes are given. The following conclusion may be drawn from this study.

Performance indicators at a signalized intersection may be analysed as a uniform and random plus oversaturation component. Formulation the uniform component of total delay may be easy since it has geometrical shape between cumulative arrivals and departures, but oversaturation component of random delay is not easy since it requires transformation expression between steady-state and time dependent queue. In order to overcome this problem, TRANSYT traffic model is a useful tool to estimate R+O in a simulation period. Transformation relation by TRANSYT also allows for the oversaturation.

UD_{model} estimates the uniform component of a delay in one step, but $R{+}O_{model}$ estimate the delay in two steps. In the first step, the R+O model is obtained when link traffic volumes is less than or equal to 290 veh/h and in the second step, the R+O model is obtained when link traffic volume is greater than 300 veh/h. The reason for solving the R+O components in two steps is due to the break point between 290 veh/h and 300 veh/h. Thus, it is not possible to obtain one form of the delay model for random plus oversaturation component.

With developed models and algorithms, performance indicators of a signalized intersection and corresponding timing parameters may be obtained via delay indexing graph without calculating many mathematical equations which may be very helpful for practitioners.

This study takes only into account four-staged four leg intersection. Changing the stage numbering and junction type to obtain delays will be carried out in future studies.

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