# LAPLACE DÖNÜŞÜMÜ

## Properties of Laplace Transform $\mathcal{L}\{f(t)\} = F(s)$

**1. Laplace Transform of Derivatives**

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\cdots \quad \cdots$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0),$$

$$n = 1, 2, \ldots$$

**2. Laplace Transform of Integrals**

$$\mathcal{L}\left\{\int_0^t \cdots \int_0^t f(u)\,(\mathrm{d}u)^n\right\} = \frac{F(s)}{s^n}$$

**3. Property of Shifting**

$$\mathcal{L}\{\mathrm{e}^{at}f(t)\} = F(s-a)$$

**4. Property of Differentiation**

$$\mathcal{L}\{t^nf(t)\} = (-1)^n\frac{\mathrm{d}^nF(s)}{\mathrm{d}s^n}, \quad n = 1, 2, \ldots$$

**5. Property of Integration**

$$\mathcal{L}\left\{\frac{f(t)}{t^n}\right\} = \int_s^\infty \cdots \int_s^\infty F(s)\,(\mathrm{d}s)^n, \quad n = 1, 2, \ldots$$

**6. Convolution Integral**

$$\mathcal{L}\left\{\int_0^t f(u)\,g(t-u)\,\mathrm{d}u\right\} = F(s)\,G(s)$$

**7. Heaviside Function**

$$\mathcal{L}\{H(t-a)\} = \frac{1}{s}\,\mathrm{e}^{-as}, \qquad \mathcal{L}\{f(t-a)\,H(t-a)\} = \mathrm{e}^{-as}F(s)$$

**8. Dirac Delta Function**

$$\mathcal{L}\{\delta(t-a)\} = \mathrm{e}^{-as}, \qquad \mathcal{L}\{f(t)\,\delta(t-a)\} = \mathrm{e}^{-as}f(a)$$

| | $f(t)$ | $\mathcal{L}\{f(t)\} = F(s)$ |
|---|---|---|
| 1. | $1$ | $\frac{1}{s}, \quad s>0$ |
| 2. | $t^n, \quad n=1,2,\ldots$ | $\frac{n!}{s^{n+1}}, \quad s>0$ |
| 3. | $t^\nu, \quad \nu>-1$ | $\frac{\Gamma(\nu+1)}{s^{\nu+1}}, \quad s>0$ |
| 4. | $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}, \quad s>0$ |
| 5. | $\cos \omega t$ | $\frac{s}{s^2+\omega^2}, \quad s>0$ |
| 6. | $t \sin \omega t$ | $\frac{2\omega s}{(s^2+\omega^2)^2}, \quad s>0$ |
| 7. | $t \cos \omega t$ | $\frac{s^2-\omega^2}{(s^2+\omega^2)^2}, \quad s>0$ |
| 8. | $e^{at}$ | $\frac{1}{s-a}, \quad s>a$ |
| 9. | $\sinh \omega t$ | $\frac{\omega}{s^2-\omega^2}, \quad s>\lvert\omega\rvert$ |
| 10. | $\cosh \omega t$ | $\frac{s}{s^2-\omega^2}, \quad s>\lvert\omega\rvert$ |
| 11. | $\sinh^2 \omega t$ | $\frac{2\omega^2}{s(s^2-4\omega^2)}, \quad s>2\lvert\omega\rvert$ |
| 12. | $\cosh^2 \omega t$ | $\frac{s^2-2\omega^2}{s(s^2-4\omega^2)}, \quad s>2\lvert\omega\rvert$ |
| 13. | $t^{\nu-1}\sinh \omega t, \ \mathcal{R}e(\nu)>-1, \ \nu\neq 0$ | $\frac{\Gamma(\nu)}{2}\left[\frac{1}{(s-\omega)^\nu}-\frac{1}{(s+\omega)^\nu}\right], \quad s>\lvert\omega\rvert$ |
| 14. | $t^{\nu-1}\cosh \omega t, \ \mathcal{R}e(\nu)>0$ | $\frac{\Gamma(\nu)}{2}\left[\frac{1}{(s-\omega)^\nu}+\frac{1}{(s+\omega)^\nu}\right], \quad s>\lvert\omega\rvert$ |
| 15. | $f(a^t), \quad a>0$ | $\frac{1}{a}F\left(\frac{s}{a}\right)$ |
| 16. | $\int_t^\infty \frac{f(u)}{u}\mathrm{d}u$ | $\frac{1}{s}\int_0^s F(u)\,\mathrm{d}u$ |

## *Properties of Inverse Laplace Transform* $\mathscr{L}^{-1}\{F(s)\} = f(t)$

**1. Property of Shifting**

$$\mathscr{L}^{-1}\{F(s-a)\} = \mathrm{e}^{at} f(t)$$

**2. Property of Differentiation**

$$\mathscr{L}^{-1}\left\{\frac{\mathrm{d}^n F(s)}{\mathrm{d}s^n}\right\} = (-1)^n t^n f(t), \quad n = 1, 2, \ldots$$

**3. Property of Integration**

$$\mathscr{L}^{-1}\left\{\int_s^\infty \cdots \int_s^\infty F(s)\,(\mathrm{d}s)^n\right\} = \frac{f(t)}{t^n}, \quad n = 1, 2, \ldots$$

**4. Convolution Integral**

$$\mathscr{L}^{-1}\{F(s)\,G(s)\} = \int_0^t f(u)\,g(t-u)\,\mathrm{d}u = \int_0^t g(u)\,f(t-u)\,\mathrm{d}u$$

**5. Heaviside Function**

$$\mathscr{L}^{-1}\{\mathrm{e}^{-as} F(s)\} = f(t-a)\,H(t-a)$$

**6. Dirac Delta Function**

$$\mathscr{L}^{-1}\{1\} = \delta(t), \qquad \mathscr{L}^{-1}\{\mathrm{e}^{-as}\} = \delta(t-a)$$

| | $F(s)$ | $\mathscr{L}^{-1}\{F(s)\} = f(t)$ |
|---|---|---|
| 1. | $\dfrac{\omega}{s^2+\omega^2}$ | $\sin \omega t$ |
| 2. | $\dfrac{s}{s^2+\omega^2}$ | $\cos \omega t$ |
| 3. | $\dfrac{1}{s^n}, \quad n = 1, 2, \ldots$ | $\dfrac{1}{(n-1)!}\, t^{n-1}$ |
| 4. | $\dfrac{1}{s^{n+\frac{1}{2}}}, \quad n = 0, 1, \ldots$ | $\dfrac{2^{2n} n!}{\sqrt{\pi}(2n)!}\, t^{n-\frac{1}{2}}$ |
| 5. | $\dfrac{1}{(s-a)(s-b)}, \quad a \neq b$<br>$\dfrac{1}{s^2-a^2}$ | $\dfrac{1}{a-b}\left(e^{at} - e^{bt}\right)$<br>$\dfrac{1}{a} \sinh at$ |
| 6. | $\dfrac{s}{(s-a)(s-b)}, \quad a \neq b$<br>$\dfrac{s}{s^2-a^2}$ | $\dfrac{1}{a-b}\left(a e^{at} - b e^{bt}\right)$<br>$\cosh at$ |
| 7. | $\dfrac{1}{(s-a)(s-b)(s-c)}, \quad a \neq b \neq c$ | $-\dfrac{(b-c)\, e^{at} + (c-a)\, e^{bt} + (a-b)\, e^{ct}}{(a-b)(b-c)(c-a)}$ |
| 8. | $\dfrac{1}{s^4+4a^4}$ | $\dfrac{1}{4a^3}\left(\sin at \cosh at - \cos at \sinh at\right)$ |
| 9. | $\dfrac{s}{s^4+4a^4}$ | $\dfrac{1}{2a^2} \sin at \sinh at$ |
| 10. | $\dfrac{s^2}{s^4+4a^4}$ | $\dfrac{1}{2a}\left(\sin at \cosh at + \cos at \sinh at\right)$ |
| 11. | $\dfrac{s^3}{s^4+4a^4}$ | $\cos at \cosh at$ |
| 12. | $\dfrac{1}{s(s^4+4a^4)}$ | $\dfrac{1 - \cos at \cosh at}{4a^4}$ |
| 13. | $\dfrac{1}{s^2(s^4+4a^4)}$ | $\dfrac{2at - (\sin at \cosh at + \cos at \sinh at)}{8a^5}$ |
| 14. | $\dfrac{1}{s^3(s^4+4a^4)}$ | $\dfrac{a^2t^2 - \sin at \sinh at}{8a^6}$ |
| 15. | $\dfrac{1}{s^4(s^4+4a^4)}$ | $\dfrac{2a^3t^3 - 3(\sin at \cosh at - \cos at \sinh at)}{48a^7}$ |

| | $F(s)$ | $\mathcal{L}^{-1}\{F(s)\} = f(t)$ |
|---|---|---|
| 16. | $\frac{1}{(s^2+a^2)^2}$ | $\frac{1}{2a^3}(\sin at - at\cos at)$ |
| 17. | $\frac{s}{(s^2+a^2)^2}$ | $\frac{1}{2a}t\sin at$ |
| 18. | $\frac{1}{(s^2+a^2)^3}$ | $\frac{1}{8a^5}\left[(3-a^2t^2)\sin at - 3at\cos at\right]$ |
| 19. | $\frac{s}{(s^2+a^2)^3}$ | $\frac{1}{8a^3}t(\sin at - at\cos at)$ |
| 20. | $\frac{1}{(s^2+a^2)(s^2+b^2)}$ | $\frac{a\sin bt - b\sin at}{ab(a^2-b^2)}$ |
| 21. | $\frac{s}{(s^2+a^2)(s^2+b^2)}$ | $\frac{\cos bt - \cos at}{a^2-b^2}$ |
| 22. | $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ | $\frac{a\sin at - b\sin bt}{a^2-b^2}$ |
| 23. | $\frac{s^3}{(s^2+a^2)(s^2+b^2)}$ | $\frac{a^2\cos at - b^2\cos bt}{a^2-b^2}$ |
| 24. | $\frac{1}{s^4-a^4}$ | $\frac{1}{2a^3}(\sinh at - \sin at)$ |
| 25. | $\frac{s}{s^4-a^4}$ | $\frac{1}{2a^2}(\cosh at - \cos at)$ |
| 26. | $\frac{s^2}{s^4-a^4}$ | $\frac{1}{2a}(\sinh at + \sin at)$ |
| 27. | $\frac{s^3}{s^4-a^4}$ | $\frac{1}{2}(\cosh at + \cos at)$ |
| 28. | $\frac{1}{s^3+8a^3}$ | $\frac{e^{at}(\sqrt{3}\sin\sqrt{3}at - \cos\sqrt{3}at) + e^{-2at}}{12a^2}$ |
| 29. | $\frac{s}{s^3+8a^3}$ | $\frac{e^{at}(\sqrt{3}\sin\sqrt{3}at + \cos\sqrt{3}at) - e^{-2at}}{6a}$ |
| 30. | $\frac{s^2}{s^3+8a^3}$ | $\frac{2e^{at}\cos\sqrt{3}at + e^{-2at}}{3}$ |